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ABSTRACT

The problem of the efficient allocation of State aid for libraries within a fixed budget constraint was studied. A management science (operations research) approach was utilized. Two basic models were developed: an allocation model to determine the amount of State aid to be received by each library system and a model that predicts library use as a function of State aid and other variables. The measure of performance used in this research as a proxy variable for library use was the number of exposure counts. These models were built, their parameters estimated, and their structure tested and validated for the case of one illustrative State, New York. Finally, it was shown how these models could be combined for policy making purposes. (Author)

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THE EFFICIENT ALLOCATION OF RESOURCES BY THE STATE TO SYSTEMS OF PUBLIC LIBARIES

Ronald M. Whitfield

University of Pennsylvania
Philadelphia, Pennsylvania 19104

May 1974

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THE EFFICIENT ALLOCATION OF RESOURCES BY THE STATE TO SYSTEMS OF PUBLIC LIBRARIES

Ronald M. Whitfield

A DISSERTATION

in

Business and Applied Economics

Presented to the Faculty of the Graduate School of Arts and Sciences of the University of Pennsylvania in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

1974

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Morris Hamburg
Supervisor of Dissertation

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Dan M. McGill
Graduate Group Chairman

ABSTRACT

The Efficient Allocation of Resources by the State to Systems of Public Libraries

Author: Ronald M. Whitfield

Supervisor: Morris Hamburg

This dissertation deals with the problem of the efficient allocation of State aid for libraries within a fixed budget constraint. A management science (operations research) approach is utilized. Two basic models are developed: an allocation model to determine the amount of State aid to be received by each library system and a model that predicts library use as a function of State aid and other variables. The measure of performance used in this research as a proxy variable for library use is the number of exposure counts. These models were built, their parameters estimated, and their structure tested and validated for the case of one illustrative State, New York. Finally, it was shown how these models could be combined for policy-making purposes.

The allocation model determined the amount of State aid that should be distributed to each library system in order to maximize a measure of library effectiveness. The allocation process was constrained by an over-all budget ceiling and equity considerations. A methodology for measuring a local area's ability to pay for library

service was devised as a part of the process of specifying the equity constraints. These ability to pay measures were contrasted with estimates of the cost of meeting a library system's minimum acceptable needs, as defined by State library administrators. A sensitivity analysis of the model showed that under certain conditions the model had multiple solutions, and that otherwise it was sensitive to changes in certain policy parameters.

The results of the prediction model were quite good. Total library use for each system can be predicted very accurately with a multiple regression model that includes only three variables: State aid, change in local government expenditures for libraries, and change in number of books added. This model explains about 96% of the variation in library use.

Finally, it was shown that the combination of the two models can be useful for policy-making purposes. Several alternative policies for distributing State aid were compared.

In conclusion, the models developed represent contributions to the process of short-term library planning and decision making. Rationalization and quantification of allocation and measurement problems of libraries for State level administrators are in the embryonic stage, and this research constitutes an advance in the state of the art in those fields.

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GLOSSARY

Except as noted, the following variables are defined for each library system.

a = coefficient for State aid in objective function; measure of effectiveness using State aid to produce document exposure.

B_{ij} = the j th tax base in library system i.

C = number of exposure counts.

$\hat{C}$ = estimated number of exposure counts.

D = population density of area served by system.

E_i = total expenditures of the i th governmental unit.

G = total local government expenditures for public libraries.

ΔG = annual rate of change of total local government expenditures for public libraries.

H = total book holdings.

K = number of books added by local purchases or gifts.

ΔK = annual rate of change in number of books added by local purchases or gifts.

L = proportion of school-age children residing in area served by system.

M = expenditures for library materials.

m = policy variable; average cost per capita of providing a minimum acceptable level of library service.

O = total operating expenditures.

P = population attributed to area served by system.

p_{ik} = average price for the k th library input in system i.

$\underset{\sim}{Q}$ = vector of variables subject to control at or below the library system level.

q_{ik} = the quantity of the k th input in system i.

r = average expenditure rate for library service for the State.

S = amount of State aid received by system.

S* = solution value for State aid using allocation model.

T = total budget available for State aid at State level.

t_j = statewide average tax rate applied to tax base j.

$\underset{\sim}{U}$ = vector of uncontrollable variables for system.

Y = potential economic resources to support library service in area served by system.

τ_i = proportion of budget of governmental unit i allocated to libraries.

τ_j = standard tax rate applied to tax base j.

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PREFACE

This dissertation deals with the question of how State administrators can efficiently allocate funds to support systems of public libraries. Two basic models are developed: a prediction model to forecast total library use as a function of State aid and other variables and a mathematical programming model to allocate State aid in an optimal fashion. These two models then are combined and are shown to be useful for policy-making purposes.

The basic stimulus and ideas for this dissertation grew out of the author's role as a member of a large research investigation of library management and statistical problems conducted under the direction and supervision of Professor Morris Hamburg and under the sponsorship of the U. S. Office of Education. This dissertation reflects and has benefited from many fruitful hours of discussion between the author and other members of the research project, particularly Leonard E. Ramist and Michael R. W. Bommer.

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"There once was a nation of 200 million people that was the most powerful country in the whole world.

"At the national level the inhabitants were very rich, but at the local level they often turned out to be quite poor. And as luck would have it, they all lived at the local level.... There were some urbanites who... stood around in small groups shaking their heads and wondering, if our country is so rich, why are the cities so poor?

"And to this day, no one has been able to answer that question."

Wilfred Owen, "An Urban Fable"

CHAPTER 1

INTRODUCTION

Introduction

The application of statistical techniques and operations research methodology in the analysis of library functions has been a fairly recent phenomenon. There is much to recommend these tools in the solution of problems faced by the library administrator. Most of the research which has been done to the present time has dealt only with individual libraries, and problems which deal with larger units of library service have been treated only in a descriptive manner. To draw an analogy from the field of economics, most of the current work has been basically micro-analytic in nature as opposed to macro-analytic. This dissertation attempts to redress this imbalance, to widen the range of tools used to analyze library problems, and to introduce new methods in the formulation and development of library resource allocation problems.

The focus of this thesis will be directed toward decisions affecting the allocation of resources for libraries at the State level. State administrators today may feel that they are handcuffed to rigid state-aid formulas which

allow little discretion in the use of funds. The parameters of these formulas need not always be the same, however. In fact, at one time decisions were made by someone to fix these parameters at certain levels, and the original conditions may have changed.

The purpose of this thesis is to develop mathematical decision and prediction models to aid State administrators to make better decisions regarding the allocation of scarce resources. Specifically, a model will be developed that "efficiently" allocates funds within a fixed State-aid budget to multi-county systems of public libraries, and criteria are proposed to give meaning to the word "efficient" in this library context. Furthermore, a model will be developed which predicts library use as a function of controllable and uncontrollable variables. This model can be used to compare the current level of library use with the predicted level of library use if State-aid were allocated according to a mathematical programming model.

It must be stated at the outset of this thesis that no mathematical decision model will supplant a manager or decision maker. The issues and problems that confront an administrator and affect his decisions are far too complex and

intricate to model completely. Many variables enter into the making of a decision; some may be quantifiable but others may not be. These latter variables might be the ones on which the decision hinges, and if this is the case, no mathematical model will be of use. Progress has been made in model building, and it is the contention of this thesis that those variables which are relevant for rational decision making are also quantifiable within reasonable limits.

Certain factors will not be considered in our decision models. For example, if an important and powerful legislator requests that certain projects be undertaken in his district, the State administrator may be strongly influenced to favor those projects. The decision models developed in this thesis will not include any variables which relate to interpersonal relationships nor political associations.

These models can be useful to the administrator in many ways. First, he can use these models to determine an efficient allocation of State funds, given the variables included in the model and any assumptions which are made. He then can process this information along with any political or other considerations to reach a final decision.

Second, he can use the models as sounding boards of this "final" decision to see the consequences of alternative allocation decisions. The administrator can thus interact with the model. Third, this interaction can be very useful for short-term planning for library programs. Various plans can be tested using these models and their efficacy ascertained. Some plans may be shown to be inferior to others and can be eliminated from further consideration. The remaining set of plans can be tested and studied until a final plan is chosen. Fourth, these static decision models can be extended to provide the basis for dynamic models in which a time frame is explicitly considered. These dynamic models would prove quite useful for long-range planning.

Resource Allocation in Private and Public Organizations

The allocation of resources in the private sector of the economy has been the subject of intensive study for about two centuries. During most of this period, it was universally assumed that the primary if not the only objective of a firm operating in the private sector was profit. Entrepreneurs, so the theory goes, would organize, direct, and mold in the most efficient manner possible all of the productive resources of labor and capital under their control to produce that level of output which would maximize

their profit.

According to the neoclassical theory of the firm, the manager of a firm whose over-all objective is to maximize profits should use two basic decision rules to guide his actions. First, the firm's optimal activity level should be that level of output or production where the marginal net yield is zero. The second decision rule which a profit-maximizing manager should follow provides guidance for the optimal allocation of resources among several activities or production processes. This rule states that the manager should allocate resources in such a way that the marginal returns per unit of effort should be equal for all activities. If resources were not allocated in this optimal manner, it is clearly possible to increase total profits given the same quantity of resources merely by reallocating these resources from activities which have low yields per unit to those activities having higher yields per unit.

These decision rules are valid for firms whose only objective is to maximize profits. From a practical point of view this may be a satisfactory approximation to reality for many firms and their managers. However, this assumption regarding profit maximization is no longer universally

accepted and, in fact, most researchers would consider it either a drastic oversimplification or else basically false. No rational manager would act without regard to profits, but neither would they act without considering other factors. As long as profits are satisfactory, managers might very well pursue other goals. Some examples of other goals which a manager might pursue are: maintaining or increasing the firm's market share, personal rewards of power and prestige, and maintaining stable labor relations. These refinements or changes in the basic assumption of profit maximization have made the economic theory of the firm more complex and have introduced psychological and other non-economic criteria into the theory. To quote from C. Michael White: "the universally accepted goal of profit maximization is in reality a heterogeneous collection of multiple goals of firms used either in place of profit maximization or in some weighted combination with it."[1] Is it any wonder that for institutions in the public sector, such as libraries, there has been so little agreement on objectives and goals?

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[1]C. Michael White, "Multiple Goals in the Theory of the Firm," in Kenneth E. Boulding and W. Allen Spivey, ed., Linear Programming and the Theory of the Firm, Macmillan Co., New York, 1960, p. 181.

In a free market economy the allocation of resources is determined by the interaction of the forces of supply and demand in the various markets and by the operation of the price mechanism. Excess demand for any product is choked off by an increase in the price for that product, and excess supply is curtailed by a decrease in price. Institutions operating in the public domain are also faced with the problem of resource allocation, but the mechanisms which guide the allocation of resources for a private firm are absent in the public sector. In libraries, for example, a reader pays no price for checking out a book. Since the library administrator acts essentially as the supplier of library resources, he is given very little indication about the depth or intensity of demand. The only data usually available to him without questioning users are the total number of documents circulated, reference questions answered, and number of persons entering the library. A businessman would be able to increase the quantity of his product demanded by reducing his price. However, a library administrator must use other tools or devices to increase demand besides lowering price: he must improve his collection, provide easier access, deliver documents quicker, or otherwise provide better service.

While a businessman can measure his success on the yardstick of profitability, there is no generally accepted comparable measure for libraries. Measures of library performance are discussed in Chapter 3.

In summary, both private and public organizations confront the problem of allocating their scarce resources to fulfill unlimited needs. The decision to direct resources in any one particular endeavor necessarily restricts the quantity of resources available for all other endeavors. This restriction is crucial to a decision maker since ideally he wants to allocate his resources in an optimal manner. His problem, therefore, is to channel the resources at his disposal into those areas in such a way that he maximizes some measure of benefit or worth. This "ideal" allocation has the property that any reallocation of resources results in a lower value on the decision-maker's benefit scale.

Public Goods Theory

The allocation of resources within a library or library system was discussed in the previous section. This section is concerned with how the library gets its resources in the first place. Since nearly all library revenue comes from

the public purse, the discussion will center on questions arising out of public goods theory, a field of economics dealing with normative issues regarding the distribution of private and public benefits and costs. Typical questions raised by this theory are: why are some goods and services privately paid for, while others are provided without charge? Should a public library charge users for its services? Who should pay for library service, and how much should they pay?

These questions are typical of the many challenging questions raised by public goods theory. Only a few such questions are addressed by this thesis, the basic ones being: why should the government, and State government in particular, contribute to the support of public library systems? How much should the various levels of government contribute?

Public goods theory has identified four major reasons for public financing of library service:

(1) underconsumption at current levels of funding

(2) external benefits to society

(3) widening individuals' opportunities

(4) changing the distribution of income.

If public libraries relied exclusively on member fees, user fees, or philanthropic gifts, a level of service would be provided which might be deemed too low for the long-term interests of both individuals and society at large. Local, State, and national governments can rectify this short-sightedness by financially supporting local libraries, thereby raising the level of service to a point commensurate with society's long-term interests.

A second reason is that the use of public libraries imparts benefits not only to individuals but also to society at large. For the individual, libraries contribute to the fulfillment of personal goals of knowledge, creativity, motivation, and confidence. However, there are certain "external" benefits or by-products of library use which accrue to the society at large. The external benefits are difficult to define, but examples are the stimulated teacher or reader, the scientific discovery, the informed voter, and the successful businessman. The State seeks to encourage and realize more of these social benefits. For efficient allocation of resources, private as well as social benefits need to be considered. The State should support libraries until the final increment to expenditure yields

a combination of private and social benefits equal to those of the final increment of expenditures for other private and public goods.

Another reason for State support of libraries is that, by providing a higher level of library service, the State can widen the opportunities of all its citizens. Low income groups particularly are affected since they may have few substitutes for library service. Widened opportunities imply a more mobile and more educated population and an increased awareness of other cultures.

The last reason is that the State may support library service in order to alter the distribution of real income. Improved service available at little or no cost enhances an individual's development and may increase his income-earning capacities. Access to a large collection of books is itself an economic good, a privilege for which some individuals would be willing to pay. Since access is provided free of charge in most cases, an individual user's real income is increased. The redistribution of income affects people at all income levels, not just at the lower end.

These are the four major reasons for State support of libraries. In practice all of these reasons tend to become

intermingled: the State simultaneously wants to encourage library use from all socio-economic classes, to capture certain social benefits associated with library use, to widen opportunities, and to alter the distribution of income.

The second question, how much should the various levels of government contribute, is a difficult question to answer. Public goods theory offers some guidance here. If libraries receive support from different levels of government only because of external benefits to society, then each level of government (city, county, State, and national) should allocate money for libraries on the basis of their perception of the benefits and costs (both public and private) of library service. Private individuals may only consider their own benefits and costs of library use, but local authorities in a town, city, or county should look at the larger problem involving the interactions of many individuals. These external benefits and costs should be calculated and factored into the allocation process. Similarly, information and ideas are not geographically restricted and there can be interactions between individuals in different cities and towns in the same State. These

interactions generate benefits and costs which should be accounted for at the State level. Finally the external benefits and costs associated with interactions across State boundaries should be accounted for at the national level. Thus each level of government should be responsible for those benefits and costs which were external to the next lower level.

Needless to say, the problems of measuring the external benefits and costs are great. Furthermore, if the other reasons for public financing of libraries are significant, then the answer to the question of how to allocate support among the levels of government becomes even more confusing. In this case public goods theory does not provide much guidance about the appropriate share of support for each level of government.

The Library and the Management Problem

The library, whether it be public, academic, or special, is a vital component of the nation's educational system. Libraries complement the educational services provided by schools, colleges, and universities, and also provide material for research, recreational, and vocational information needs. Libraries serve as the storehouse for

the nation's growing document resources, but they are not limited to merely archival functions. Libraries can be dynamic collections of printed books and periodicals, films, records, microforms, and other types of document resources. In order to be dynamic, libraries have to change with the times. They must not only be aware of new document forms but must also take the pulse of their clientele in order to ascertain trends in their actual and potential demands.

In addition to being involved in changes in library operations and services, libraries are influenced by the cross-currents of our modern technological society. Demands on library services will increase in the future for many reasons. The body of knowledge expands daily at an increasing rate while at the same time our population is growing and the educational level of the nation is getting higher and higher. Furthermore, we are experiencing an "information explosion" resulting in a proliferation of books, journals, government, business and professional association reports, manuals, abstracts, and catalogs.

These changes have broadened man's horizons, widened old fields of endeavor and created new ones too. Frag-

mentation of knowledge has led to intense specialization, and libraries have been severely challenged to meet present-day demands. The pressures put on library services portend new approaches to solve the libraries' problems. In the short run these new approaches may consist of applications of computer technology and data processing equipment, together with more extensive use of microform techniques, to existing library functions. In the long run, however, technological developments not even presently perceived may generate more radical changes in our current conceptions about libraries. While the basic functions of a library will remain the same, the methods of carrying out these functions in the future could conceivably only involve interactions between man and machine without the physical presence of a library.

The Hierarchy of Library Service

If we conceive of library planning and decision making as being built on a hierarchy of levels, each of which is concerned with increasingly larger units of service, then each of these levels must consider and decide how to allocate the resources under its control. The smallest unit of service is the branch, any number of which can serve a local

community. Individual local libraries can join together to form systems, and systems of libraries fall under the supervision of a State library agency. At the top of the hierarchy are the various agencies and organizations that comprise the planning and decision making apparatus at the national level. This structure is shown in Figure 1.1. The discussion that follows first considers the national level of library service, then the systems level, and finally the State level. The reason the sequence does not systematically follow the order of Figure 1.1 is that only a brief discussion of the national and systems level of library service will be presented in this chapter, and the remainder of this thesis will focus on the State level.

National Level

At the national level there is no single, unified decision-making body which both establishes national objectives and policies for library development and has broad responsibilities for carrying out these policies.

Generally there are three major centers for library decision making and library leadership within the Federal government--the National Libraries, the Congress, and the Office of Education. Outside of government a number of

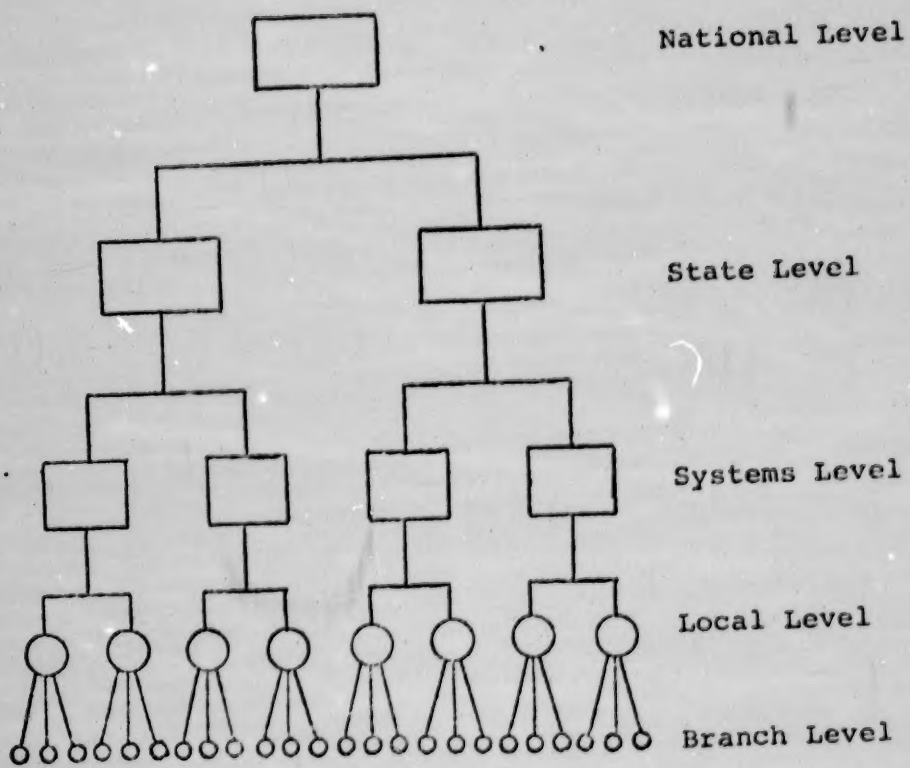

Figure 1.1: Hierarchy of Library Service

national professional organizations exist for the promotion of library interests, for helping to establish national goals and policies, and for carrying out research into library problems. Some examples of organizations in this category are the American Library Association, the Council on Library Resources, and the Association of Research Libraries. All of these organizations and agencies can be considered to be national library leaders since library planning and decision making at the national level results from their collective efforts and influence. No one organization is dominant.

National Libraries: The National Libraries are the Library of Congress, the National Library of Medicine, and the National Agricultural Library. The Library of Congress is the most famous of the three, and it has become "recognized as the most important and the leading library in the country."[2] It performs most of the functions generally reserved for a national library, but it is not the National Library. Although it is the largest library in the country in terms of number of documents, it does not have a comprehensive national collection. Hence it must share the title

[2]Douglas M. Knight and E. Shepley Nourse, eds., Libraries at Large, R. R. Bowker, New York, 1969, p. 349.

of "National Library" with the other two libraries, the National Library of Medicine and the National Agricultural Library, whose function it is to serve as resource depositories in the fields of medicine and agriculture, respectively.

Congress: The role of Congress in the national library picture has been rather unique. Ordinarily it would have been the duty of the Executive branch of the government to propose new programs of Federal aid for libraries. However, "library-aid legislation has come about chiefly at the initiative of Congress, or perhaps better put, at the initiative of lobbyists active in behalf of libraries, rather than as part of any Executive program or drive."[3] The principal piece of legislation which provided aid for public libraries was the Library Services Act of 1956, but the provisions of the law precluded aid to all but rural areas. In 1964 the program of aid for public libraries was broadened to include all libraries, whether rural or urban. At the present time Federal aid under the Library Services and Construction Act (LSCA) is being used for library construction, interlibrary

[3] Ibid., p. 369.

cooperation, service to the physically handicapped and institutionalized, and to provide library service in areas inadequately served and in areas with a large percentage of disadvantaged persons. Other library-aid legislation has been aimed at school libraries (Elementary and Secondary Education Act of 1965, Title II), college and university libraries (Higher Education Act of 1965, Title II-A), and library research and training (Higher Education Act, Title II-B).

Office of Education: The Office of Education, an agency within the Department of Health, Education, and Welfare, is the third major organization on the Federal library scene. In many respects the Office of Education is the most important organization, since it is responsible for the administration and implementation of the bulk of the various legislative programs pertaining to libraries. To a large extent it also contributes importantly to the setting of national policies for libraries.

The goals of the Office of Education for carrying out its programs for library development are:

(1) To expand and extend library programs and services for the disadvantaged and handicapped.

(2) To provide greater equalization of State and local resources for library programs and services.

(3) To promote better utilization of library resources including cooperation and coordination among public, school, academic, special, and private libraries.

(4) To improve generally the quality of library programs.

These goals have now become incorporated as statements of policy in recent library-aid legislation, but this does not ensure their fulfillment. The Office of Education's basic problem has been its lack of authority over how the funds are spent. For example, the 1970 Amendments to the Library Services and Construction Act (LSCA) declare that "It is the purpose of this Act...to afford the States greater discretion in the allocation of funds...to meet specific State needs." Furthermore, "the determination of the best uses of the funds provided under this Act shall be reserved to the States and their local subdivisions."[4]

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[4] From correspondence with Ray Fry, Director, Division of Library Programs, to R. M. Whitfield, December 8, 1970.

The Office of Education's role in LSCA as well as in the other grant programs it administers is minimal. It is given little control in the use or allocation of Federal funds, except in the broad sense that the State's expenditures must be consistent with the approved State plan. In the judgment of at least one investigator, "the kindest thing that can be said is that the Office of Education... is a convenient and mindless conduit for channeling tax funds to public, school, and college libraries. Its methodical calculation of grants by arithmetic criteria keeps the system clean of politics - and intelligence."[5]

The lack of the essential element of control over allocations and expenditures distinguishes decision making within the Office of Education from normal, profit-oriented decision making. Of course, the Office of Education is certainly not unique within the public sector in this regard. In fact many would argue that it is not the function of the Federal bureaucracy to be concerned with individual local public investment projects since officials at the State level are in a better position to judge their worthi-

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[5]Harold Orlans, "Observations on Government Library Organization and Policy" in Douglas M. Knight and E. Shepley Nourse, eds., Libraries at Large, p. 390.

ness in the light of local needs. Furthermore, State and local officials can be held more directly accountable for a particular program's success or failure than a Federal bureaucrat. The Federal government, so the argument runs, should limit itself to functions it is better equipped to handle.

The precise role of the Office of Education is therefore somewhat nebulous, and its function in the distribution of library-aid funds may be preempted entirely under Federal revenue sharing plans. Under the current plans all library categorical aid programs would be replaced by special revenue sharing, and libraries would have to compete with other State educational institutions for funds.

Whatever may be the political fate of revenue sharing, decision making about library programs as well as education seems destined to be focused more and more at the State level. The trend at the Federal level has been to distribute funds in some equitable fashion to the States, and then to give the States increasingly more control over the final distribution of these funds. It is essential, therefore, for State level administrators to have a rational planning and decision-making framework for efficiently allocating these scarce resources.

Library Systems

The intermediate level in the hierarchy of library service is composed of individual libraries agreeing to cooperate and form library systems or networks. No one has successfully defined either of these latter two terms, but they include almost any type of cooperative arrangement which can be made between two or more independent libraries. These arrangements can be formal, legally-binding contracts as well as less formal, even informal, agreements to cooperate. Library systems can have a unified governing board, or they can be led by the head librarians of the component libraries. There is great diversity in these arrangements, and currently no one form of cooperation has taken precedence.

Library cooperation is the principal method that has been developed to meet the multifaceted pressures of current library users. In fact,

> "if a single trend stands out most clearly as a development of present times, it is probably the condition which has been characterized as 'calculated independence.' No agency or individual operates alone or independently any longer. The keynote of social progress lies, in

> fact, in successfully relating individual efforts to mutual or overlapping interests."[6]

Library systems have many dimensions, and the structure of one's analysis and opinion depends on which particular dimensions are chosen for study. For example, one dimension which received attention by the Nelson Associates is the type of governmental unit supporting public library systems. In their survey of public library systems, the criterion used for inclusion was "systems with more than one outlet open at least ten hours a week with paid staff and providing library service across political jurisdiction lines."[7]

The Nelson Associates found 491 multi-jurisdictional library systems satisfying this criterion. These systems followed five predominant patterns:[8]

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[6]New York (State), Division of Evaluation, Emerging Library Systems: The 1963-66 Evaluation of the New York State Public Library System, University of the State of New York, The State Education Department, Division of Evaluation, Albany, 1967, p. 226.

[7]Nelson Associates, Inc., Public Library Systems in the United States: A Survey of Multijurisdictional Systems, American Library Association, Chicago, 1969.

[8]Nelson Associates, Inc., Public Libraries in the United States: Trends, Problems and Recommendations, Report prepared for the National Advisory Commission on Libraries, New York, 1967, p. 19.

(1) County Library

(2) Multi-county or regional library

(3) Special district library

(4) State supplementary library

(5) State-wide, State-governed library

Each of these five basic patterns of library system development may have several different structural forms.

While certain patterns of library systems may be popular or prevalent at this time, it is unlikely that any one particular pattern will become universally accepted. Similarly, in terms of financial arrangements among cooperating libraries and in terms of the number and type of services these libraries offer, there is a superabundance of different library systems.

The problems of resource allocation in systems of libraries are quite important. However, these cooperative arrangements can become very complex. Progress for future research on these problems depends upon a clear and useful definition of the concept "library system."

State Library Agency

The level in the hierarchy of library service which provides the interface between local public libraries and

library systems with the national level is the State Library Agency. The organization of this agency differs from State to State, and it is not always called by the same name. In fact it may not even be one single agency. No matter how it is organized nor how it is named, the functions which a State library agency performs are fairly well defined.

Basically these functions are a diverse conglomeration of traditional public library functions, special library functions, and functions related to library development and library finance. A useful seven-fold classification of State-level library functions is:[9]

1. Historical library services
2. Law library services
3. Legislative reference services
4. Document depository services
5. General library services
6. Library development services
7. Others (e.g., services to the handicapped, institutionalized)

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[9]Joel Williams, Library Statistics: A Handbook of Concepts, Definitions, and Terminology, American Library Association, Chicago, 1966, p. 62.

These seven principal functions of State library service are broadly related to the needs and responsibilities of State government. The first three of these functions are quite specialized. The fourth function, document depository services, is an archival function intended to satisfy the State government's need for storing official documents.

In addition to these services the State also provides general library services for the use of both the government and its agencies and the public at large. In many cases there is a separate library, called the State Library, which fulfills this role. Otherwise, this function may be performed by a large public library within the State or by the library of a State-supported university. A major purpose of providing general library services is to serve as a resource backstop for the State's public libraries.

Probably the most rapidly expanding function of the State library agency is its library development services. As indicated in the preceding section dealing with the Office of Education, the State library agency has been given increasing importance in the administration and distribution of Federal funds for libraries. In fact, specific provision

has been made for Federal support of library development activities at the State level in recognition of the administrative costs of federal grant programs.[10] Most States have reacted positively to this Federal stimulus and have augmented the Federal funds with State grants for library programs. A fuller discussion of this topic is deferred to Chapter 2.

Among the other services provided by the State library agency are specialized activities performed for segments of the population unable to utilize regular library facilities or resources. The primary categories of these activities are: library services for the inmates of State prisons and correctional institutions; library services for the mentally ill and residents of State schools and hospitals; and library services for the blind and physically handicapped.

Summary of Major Findings

This research has resulted in four major findings or contributions. They can be classified as:

(1) a methodology for constructing ability to pay measures for library service

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[10] 45 CFR 4.1 sec. 130.86.

(2) a model for predicting library use

(3) a model for allocating State aid efficiently

(4) a combination of the prediction and allocation models for planning purposes.

These findings will be discussed briefly below.

The first contribution is the development of a methodology for constructing ability to pay measures for library support. Measures of fiscal capacity and effort for local areas have been proposed previously, but never for a specific type of government service. This research extends past efforts, and the measures that are produced are necessary and important ingredients in the equitable allocation of State aid.

The second finding is the formulation of a model for predicting library use as a function of three variables: State aid, change in local government expenditures for libraries, and change in number of books added by local purchases and gifts. This model was tested and proved to be an extremely good predictor of library use.

The third finding is the development of a linear programming model for allocating State aid. The objective in this model is to maximize a measure of library effective-

ness, subject to a budget constraint and equity constraints. The decision variables are the amount of State aid allocated to each library system. The model was tested using alternative assumptions for the coefficients in the objective function, for policy variables that enter the equity constraints, and for the budget constraint. Under certain conditions for the objective function, the model had multiple solutions. In addition, the model was sensitive to changes in these coefficients and to changes in the equity constraints.

The fourth finding is that the combination of the prediction and allocation models can be quite useful for planning purposes. For example, it was found that the models are more error-prone for urban library systems than for suburban or rural systems. Several policies for distributing State aid were formulated and examined with these models.

In this chapter we have briefly introduced the central problem to be addressed by this thesis and have discussed resource allocation for private firms and public organizations. We then looked at the hierarchy of library service and described the levels in this hierarchy above the level

of the individual library. We finally focused on the State library agency and the functions that it performs. In Chapter 2 we will describe in greater detail the State's role in library development activities, particularly with reference to New York State.

Chapter 3 will review State library objectives and performance measures and will develop the framework for the model for the efficient allocation of State aid. Chapter 3 will also present the results of the prediction model. Chapter 4 will discuss the mathematical formulation, estimation, and solution to the allocation model, and the sensitivity of the model to changes in the parameters. Finally, Chapter 5 will discuss implications, conclusions, and suggestions for future research.

CHAPTER 2

THE STATE'S ROLE IN LIBRARY DEVELOPMENT

This chapter deals with the objectives and functions of the States in relation to the growth and development of public libraries within their borders. The great diversity of statewide library development agencies and activities presents problems of comparability among the States, but one common problem for all States is the allocation and administration of Federal and/or State grants-in-aid for library development. State-aid formulas will be critically analyzed in this chapter, and formulas in selected states will be presented. Finally, New York State will be used as an example of how one State perceives its role in library development activities and how it contributes to the support of its public libraries. New York is one of the nation's leaders in these areas.

Recent Trends

Under the impetus of recent Federal legislation, the State's role in the growth and development of statewide library service has been steadily increasing. Not only have Federal funds increased over the past fifteen years, but these funds have produced a groundswell of State sup-

port for library service. Most States have accepted this financial responsibility and have pushed forward their plans for statewide library development. The Federal government's response during this period of time from 1956 to 1971 has been to recognize the crucial position of the States in developing effective library service and, as was pointed out in Chapter 1, the States have been accorded almost absolute control over the distribution of funds in Federal programs.

Not only has the Federal government looked to the States for assistance in carrying out programs of financial support of libraries, but local governments as well have looked to the State for relief of their pressing problems, many of which could be alleviated by monetary infusions. This trend toward State-funded service systems has been described as:

> One of the potentially most important developments during the past ten to fifteen years.... One reason for describing these developments as important is that... several of the efforts have been successful in shifting the financial base of local libraries to a considerably greater extent than have most other devices.[1]

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[1]Ralph Blasingame and Ernest R. DeProspo, Jr., "Effectiveness in Cooperation and Consolidation in Public Libraries," in Melvin J. Voigt, Advances in Librarianship, I, New York, Academic Press, 1970, p. 194.

The State library agency has become the pivotal point for the dual forces of: (1) Federal government seeking to decentralize its library development programs under the aegis of State plans, and (2) local libraries and library systems searching for improved methods of funding and service and a wider basis of support. These twin forces portend an ever-increasing role of the State.

In conclusion, "a combination of forces having profound implications for library development of all kinds have forced a recognition among library leaders of the need, particularly at the national and state levels, for financial support, for services, and for leadership which cuts across...geographic and political boundary lines."[2] And the State library agency is uniquely qualified to fill this position. The State serves as a natural geographic and political boundary which defines in most cases a reasonable service area. "It is the state that has the power to establish and raise standards of service. It is the state that can equalize differences in the economic resources between the various sections within its jurisdic-

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[2]S. Gilbert Prentiss, "State Libraries: The State as Producer of Library Statistics," in David C. Palmer, ed., A National Plan for Library Statistics: Guidelines for Implementation, Statistics Coordinating Committee of the ALA, Chicago, April, 1970, p. 119.

tion.... It is the state that can invoke and bring to bear a wide range of sources of tax revenue,"[3] and it is the State that already has the machinery for distributing these resources. Finally, the education of all of its citizens historically has been a responsibility of the State and public libraries are part of the educational process. Whether willingly or unwillingly, the States increasingly have been called upon to bear the burden of library leadership and support.

The extent and pattern of State financial support of public library service varies a great deal. Some states contribute heavily while others make only minimal efforts at State aid. Table 2.1 shows the pattern of financial support for large public libraries from the three principal sources of funds in 1968, ranked in order of the State with the largest relative amount of State aid. Table 2.2 presents data showing the actual financial contributions from the three principal sources to large public libraries, ranked in order of the thirteen States which contribute the largest absolute amount of State aid. Table 2.3 presents the data from Table 2.2 on a per capita basis. The data

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[3]Douglas St. Angelo, A. M. Hartsfield, and H. Goldstein, State Library Policy, Chicago, American Library Association, 1971, p. 108.

from all three tables refer only to public libraries serving at least 25,000 inhabitants.

The data in Tables 2.1, 2.2, and 2.3 show that some States have established substantial library support programs. Hawaii, for example, operates a statewide library system financed entirely by State and Federal sources. However, this is unusual and the pattern for most States has been to rely on local tax revenues for between 70% and 90% of total operating fund receipts. The aggregate United States data indicate that in 1968, 83.1% of public libraries' total operating fund receipts came from local taxes, 5.9% came from the State, 2.6% from the Federal government, and 8.4% came from other sources.

Although many States have established substantial support programs, some States do not have a grant program at all. A total of eleven states reported that no State funds were contributed to public libraries' operating funds in 1968, and of the thirty-seven States which did support public libraries, contributions ranged from Maine's $600 to New York's $7,000,000.[4] It should be pointed out that these

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[4]U. S. Department of Health, Education, and Welfare, Statistics of Public Libraries Serving Areas with at Least 25,000 Inhabitants - 1968, Government Printing Office, Washington, D. C., 1970, p. 10. Two states are not included: Georgia, a non-respondent State, and Alaska,

Table 2.1

Percentage of Operating Funds by Source for Public Libraries Serving at Least 25,000 Inhabitants, 1968 - Selected States

State	% Operating Receipts from		
	State Govt.	Local Govt.	Federal Govt.
Hawaii	89.0	0.0	11.0
Pennsylvania	18.7	62.8	4.4
Maryland	14.7	78.6	3.2
South Carolina	13.1	70.6	11.8
New Jersey	12.9	78.6	4.1
North Carolina	11.3	71.1	9.3
New York	10.9	67.3	1.6
Florida	7.3	73.1	3.1
Illinois	6.8	87.1	0.0
Rhode Island	4.9	61.2	0.0
Massachusetts	4.6	90.3	0.8
Montana	4.4	92.7	0.0
Michigan	4.4	82.5	1.3
Aggregate United States	5.9	83.1	2.6

Source: U.S. Department of Health, Education, and Welfare, Statistics of Public Libraries Serving Areas with at Least 25,000 Inhabitants - 1968, Government Printing Office, Washington, D.C., 1970.

Table 2.2

Receipts into Operating Funds by Source for Public Libraries Serving at Least 25,000 Inhabitants, 1968 - Selected States

State	Operating Fund Receipts		
	State Govt.	Local Govt.	Federal Govt.
New York	$7,072,531	$43,767,783	$1,061,798
Pennsylvania	3,956,308	13,756,816	958,146
Hawaii	2,506,560	-0-	308,497
New Jersey	2,373,396	14,498,085	765,685
Maryland	2,211,623	11,854,951	483,776
Illinois	1,264,691	16,183,102	-0-
Michigan	862,327	16,055,415	257,285
Massachusetts	824,833	16,018,573	138,060
Florida	657,166	6,537,134	274,461
North Carolina	644,382	4,041,129	527,019
California	439,808	63,465,522	754,509
Missouri	431,071	10,631,860	456,785
South Carolina	316,565	1,707,569	286,220
Aggregate United States	25,493,389	359,067,898	11,234,374

Source: U.S. Department of Health, Education, and Welfare, Statistics of Public Libraries Serving Areas with at Least 25,000 Inhabitants - 1968, Government Printing Office, Washington, D.C., 1970.

Table 2.3

Per Capita State Aid and Total Operating Receipts Per Capita to Public Libraries Serving at Least 25,000 Inhabitants, 1968 - Selected States

State	State Aid Per Capita	Total Operating Receipts Per Capita
New York	$0.54	$4.98
Pennsylvania	0.43	2.34
Hawaii	3.96	4.45
New Jersey	0.55	4.26
Maryland	0.74	5.03
Illinois	0.23	3.36
Michigan	0.15	3.43
Massachusetts	0.25	5.32
Florida	0.19	1.14
North Carolina	0.16	1.42
California	0.03	4.60
Missouri	0.14	4.00
South Carolina	0.14	1.07

Source: U.S. Department of Health, Education, and Welfare, Statistics of Public Libraries Serving Areas with at Least 25,000 Inhabitants - 1968, Government Printing Office, Washington, D.C., 1970.

data refer only to public libraries serving areas with at least 25,000 inhabitants. State support for libraries serving less populated areas and for other statewide programs often can be substantial. For example, New York's support of its larger public libraries in 1968 is more than matched by the total amount of State aid, which amounted to $14,500,000 in the same year.[5] The manner in which most States in general and New York State in particular distribute their aid will be the subject of a subsequent section.

Fair-Share and State-Aid Formulas

There have been frequent discussions in the literature about an "equitable" division of financial support among the three principal levels of government, local, State, and Federal. The question has usually been phrased, "What is a fair share for State and Federal support of local library programs?" The question has a long history of contradictory answers.

> In 1948, with Amy Winslow, (Carleton) Joeckel proposed a formula for public library support of 60% local, 25% state and 15% federal. At an Allerton Park Institute in 1961,

whose eligible libraries did not respond.

[5] Public and Association Libraries Statistics - 1968, The University of the State of New York, The State Education Department, New York State Library, Division of Library Development, Albany, 1968, p. 3.

> Hannis Smith proposed a support formula of 40% local, 40% state and 20% federal. In 1964, Lowell Martin proposed as a reasonable ratio 50% local, 30% state and 20% federal. All of these proposals are in striking contrast to the actual pattern of support in 1964, which was 81.8% local, 8.6% state, 2.1% federal and 7.5% from library fines and endowments.[6]

However, it is not likely that any "fair-share" formula will suffice. The concept of "fair-share" is couched in the presumption that local library service can and should be supported by higher levels of government. The reasons behind this presumption were discussed in the context of public goods theory in Chapter 1. The reasons given are neither mutually exclusive nor exhaustive, but the issue of a "fair-share" formula must be viewed in a political and social, as well as economic context. A "fair-share" formula is the product of innumerable value judgments, and there can be no efficient or unobjectionable method for choosing between different sets of values. Thus the division of the total costs of library service depends upon the state of the economy, the political climate at the time, and the shifts and changing demands of the social structure.

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[6] Nelson Associates, Inc. American State Libraries and State Library Agencies: An Overview with Recommendations, Report prepared for the National Advisory Commission on Libraries, New York, 1967, p. 17.

While there has been frequent discussion and a notable lack of consensus concerning a "fair-share" formula, State-aid formulas have received very little attention at all. To a certain extent this can be attributed to the quasi-legal status of these formulas, as well as their complexity and lack of uniformity nationwide. Most States which have a substantial library aid program have solved the economic and political controversies of how and where to allocate State aid by incorporating a distribution formula in the legislative fabric of the State.

The fundamental problem which is raised by the quasi-legal and rigid status of a State-aid formula is that conditions change. The factors which influenced the development of the present formula may have changed, yet there often is no mechanism for altering the formula. As Ralph Blasingame incisively points out, "there has been very little interchange of ideas on how to establish a formula for state aid or, more importantly, whether a state aid formula should or can affect the basic conditions of public library service."[7] He continues

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[7] Ralph Blasingame, "A Critical Analysis of State-Aid Formulas," Library Trends, XIX (October, 1970), p. 250.

> A state aid formula is usually...one of the principal end results of a study of the conditions of library service. The results of the formula would probably be better if they were the result of a long-range, open-ended process of planning and thus perhaps open, by definition, to change. However, the fact is that most formulas are established following a relatively short period of study, and for a variety of reasons they become difficult to change. What should be a process then becomes an event, most often under the control of persons or groups of persons who do not carry responsibility for implementing the plan.[8]

Examples of State-aid formulas are presented below for California, Illinois, Pennsylvania, and New York.

Formulas in Selected States

California

California's State-aid formula consists of two basic components: annual per capita grants and establishment grants. Annual per capita grants are made to library systems on the basis of the ratio of the statewide average assessed taxable property valuation per capita to each system's assessed valuation per capita. Sparsely populated areas are eligible to receive supplemental per capita grants by means of an adjustment in the above ratio.

Establishment grants shall be no more than 25 percent of the total operating expenditures for the library or

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[8] Ibid., p. 250.

library system for the previous fiscal year. These grants are only awarded for two years and are intended to encourage libraries to join a system. The amount of an establishment grant shall not exceed $10,000.

To qualify for State aid, libraries must meet certain minimum standards of service. These standards differ according to the population density of the library's service area. Some of these requirements are:

(1) Equal legal access to all residents of the area.

(2) An annual increase of at least 4,000 new book titles if the population per square mile is less than 75; otherwise an increase of at least 7,000 new book titles.

(3) An annual purchase of at least 400 periodical titles if the population density is less than 75; otherwise at least 700 periodical titles.

(4) An annual expenditure from local sources for total operating expenses at least as large as either 1.5 mills of total assessed valuation or $2.50 per capita.

Illinois

The State-aid formula for Illinois is composed of three factors: annual per capita grants, establishment

grants, and annual equalization grants. The per capita grants consists of two parts: an amount of $0.40 per capita for the population of the area served plus an amount based on the number of square miles and the number of counties in the service area. If the library system serves only one county, it receives $5.00 per square mile served, and if two or more counties are included in the system, then $15.00 per square mile is granted.

Establishment grants are given to library systems only once. Upon the approval of the system by the State Librarian, a grant of $25,000 is made to a system serving one county in whole or in part. For each additional county or part of a county served by the system, an additional grant of $15,000 is made.

In order to qualify for an annual equalization grant, local taxing districts must levy a tax for library purposes of not less than .6 mills of the full assessed valuation of all taxable property. The State-aid formula sets a floor of $1.50 per capita of local tax revenues, and if a qualified library receives less than $1.50 per capita in local tax revenues for libraries, then the State Librarian is authorized to make an equalization grant based upon the

difference between $1.50 per capita and the amount obtained from a tax rate of .6 mills.

Pennsylvania

Pennsylvania's State-aid formula recognizes three types of libraries, each representing a larger service area: local public libraries, district library centers, and regional library resource centers. State aid to these libraries consists of annual per capita grants and equalization aid.

In order to qualify for State aid, local public libraries must annually expend funds in excess of either

(a) .5 mills times the market value of real estate in the library's service area, or

(b) $2.00 per capita.

Each library which qualifies for State aid is eligible to receive an annual grant of $0.25 per capita. Equalization aid in Pennsylvania is given to any library where .5 mills times the market value of real estate is less than $1.25 per capita. The amount of such aid is the difference between these two quantities.

District library centers serve other public libraries within their service areas, and they are allocated annual

per capita grants for this purpose. If the district library center is a public library, it is eligible to receive an additional $0.25 per capita for the population it serves over and above its own population. If the district library center is an academic library, it receives $0.125 per capita for the population of the district.

Regional library resource centers provide library services to district centers and local libraries and are designed to have strong research collections. There are four regional library resource centers in the State, and they are each eligible to receive up to $100,000 annually to develop their collections.

New York

The State-aid formula for New York consists of the following factors:

(1) Annual per county grant

(2) Annual per capita grant

(3) Book reimbursal factor

(4) Square miles factor

(5) Central Library Development Aid

(6) The Research Libraries of the New York Public Library grant.

State aid is allocated in New York not to individual public libraries but rather to qualified library systems (sometimes these may be coterminous). In order to be eligible for State aid, the library system must submit an approved plan of service and must expend from local tax sources an amount not less than the average amount expended during a base period.

The annual per county grant is given to library systems based upon the following criteria: if the system serves less than one county, it receives $10,000; if the system serves one entire county, it receives $15,000; if the system serves more than one county, then it receives $20,000 for each entire county served plus $10,000 for each county served in part.

The per capita factor of the State-aid formula amounts to $0.40 per capita of population of the area served.

The book reimbursal factor rewards library systems which expend more than $0.40 per capita for books, periodicals, and binding. The excess amount spent the preceding year shall be returned to the library system through State aid, provided that the total amount returned is less than $0.30 per capita of population served.

The square miles factor in the State-aid formula depends on the number of counties served by the library system. For a system composed of one county or less, State aid shall amount to $8.00 per square mile of area served. For a multi-county library system, the sum of $8.00 per square mile shall be augmented by $4.00 per square mile for each additional entire county served, provided that the total square mile factor shall not exceed $24.00 per square mile of area served. If one county is served by two or more systems, a pro rata share for each system is calculated based on population.

Any system which submits a plan for the further development of its central library shall be entitled to receive an additional grant equal to the larger of:

(a) $0.05 per capita of population of area served, or

(b) $15,000.

Furthermore, special recognition is given to The New York Public Library and the importance to the State of its central reference department and book collection. Its unreimbursed expenditures on books, periodicals, and binding shall be covered by State aid as long as such expenditures are less than the maximum allowable under the formula - $525,000.

Limitations of a Formula Approach

As the examples above indicate, State-aid formulas are composed of many different factors which are weighted in different proportions. The major common factors of most State-aid formulas are:

(1) grants based upon population served

(2) grants based upon area served

(3) grants intended to equalize resources.

In addition, most formulas explicitly take into account the particular institutional environment of the State (e.g. New York's formula recognizes the unique position of New York City's Public Library in the statewide system).

Reliance upon a simplistic formula to allocate State aid can lead to serious inequities and it is important to recognize the limitations of the formula approach. Ralph Blasingame in a previous quote identified one glaring limitation of most formulas: what should in reality be a process of balancing State aid against local conditions has in actuality solidified to become an event.

This situation would not be so serious except for the fact that there are no incentives in the formula approach to recognize changes in the environment. A formula

is simply a distributive tool which, once it becomes established, has a tendency to become inviolate. Efforts by some libraries to change the formula would naturally run into the political opposition of other libraries which might be threatened by such changes. The forces for change thus must overcome the usually stronger forces which resist change. Instead of gradual shifts in the manner in which State-aid is distributed, major battles must be waged to effect changes.

The greatest criticism of the formula approach as practiced by most States is the informational sterility of the methodology. There are five major methodological flaws in the approach:

(1) the formula approach offers no measure of the extent to which the State's objectives for library support and development are achieved. Funds are distributed, but there is no measure of effectiveness.

(2) Few factors are usually included in the formula. A parsimonious formula is indeed an admirable goal, but not when it is achieved by a distortion of stated objectives. All essential factors need to be considered when State aid is distributed.

(3) There is no analysis in State-aid formulas indicating how the various factors affect library service. For example, what is the relationship between the number of square miles which a library serves and the level of State funding for that library? Furthermore, how is this relationship affected by population density and number of service outlets? The factors affecting library service are complex and are probably not independent. In order to produce a more efficient allocation of resources, these complex interrelationships need to be studied, and this requires more sophisticated techniques than a simple formula can provide.

Considering the first three limitations above, the next two limitations are less significant although they cannot be overlooked.

(4) Short-term and long-term planning for library service are important functions generally performed by State library agencies. One aspect of these activities is to assess the impact of various possible levels of State funding, not

only in terms of which libraries receive how much money but also which population groups are affected, what the incremental effect on total library service will be, and what the incremental effect will be for individual libraries or groups of libraries. These types of questions are necessary to plan skillfully for future library service and to operate effectively in the present. The formula approach is not very useful as an analytical tool in this regard. Given a fixed budget, a formula can answer the question: how much money goes to each library system. Unfortunately its usefulness for other types of questions is severely restricted. For example, it does not indicate how much total library service would decrease if the budget were to be cut by $1 million, or which particular libraries would be the most severely affected by such a budget cut.

An additional aspect of library planning is to study and judge the merits of alternative allocations of State aid. Without a useful measure of output or performance, a formula approach is

meaningless in this context. Library planners and administrators need to make decisions such as which special projects should be supported this year, or how should the State-controlled Federal grants be allocated in the most efficient manner.

(5) The final limitation of the formula approach is its lack of usefulness as an incentive tool. State aid need not be looked upon as a carrot to dangle before the watering mouths of local librarians, but responsive and efficient library management should be rewarded. The formula approach allocates funds regardless of performance, and thus it offers no incentives for good service. There are very few professional managers today who would maintain their positions unless their performance were satisfactory, and good performance should also be a criterion for library managers.

New York State Library System

For illustrative purposes, the system of public libraries in New York State will be studied in great detail.

In the preceding section, the factors utilized in New York to allocate State aid were discussed. In this section, we will describe briefly this State's system of public libraries.

New York has a well-developed and highly-regarded system of public libraries. Annual appropriations to its library systems are the highest in the nation, and the State ranks among the highest for State aid per capita. The annual budget for State aid has increased considerably over the last ten years, spurred in part by increased levels of Federal library support. From 1961 to 1970, State aid grew from $6.4 million to $14.3 million.

In 1969 there were 705 chartered public libraries serving over 99 per cent of the State's population. Each of these individual libraries is a member of one of twenty-two library systems. Figure 2.1 presents a map of New York State indicating the location of each library system. Table 2.4 lists the twenty-two systems and the population and area served by each system.

Library systems in New York are geographical entities. With one exception, a system is composed entirely of one or more of the State's counties. Ulster County is the only

Figure 2.1: LOCATION OF PUBLIC LIBRARY SYSTEMS IN NEW YORK STATE

Table 2.4

Population and Area Served for
New York Library System Areas

System Name	Population Served, 1970 Census	Land Area Served
Brooklyn	2,601,852	76
Buffalo and Erie	1,113,491	1,054
Chatauqua-Cattaraugus	228,971	2,415
Chemung-Southern Tier	284,109	3,543
Clinton-Essex-Franklin	151,496	4,570
Finger Lakes	281,808	2,547
Four County	369,082	4,101
Mid-Hudson	487,448	2,974
Mid-York	403,341	3,330
Mohawk Valley	294,249	1,740
Nassau	1,422,905	251
New York	3,292,200	125
Nioga	331,747	1,430
North Country	325,040	6,321
Onondaga	472,185	792
Pioneer	961,899	3,165
Queens	1,973,708	113
Ramapo Catskill	520,480	2,508
Southern Adirondack	228,520	4,281
Suffolk	1,116,672	922
Upper Hudson	438,128	1,196
Westchester	891,409	435
Total	18,190,740	47,889

Source: U.S. Department of Commerce, Bureau of the Census, 1970 Census of Population, Government Printing Office, Washington, D.C., p. 3; The University of the State of New York, The State Education Department, New York State Library, Division of Library Development, Public and Association Libraries Statistics/1969, Albany, N.Y., 1969, p. 2.

exception. Ulster is divided into two sections, one of which is part of Ramapo-Catskill Library System, the other being a member of Mid-Hudson Library System. The part of Ulster County belonging to Mid-Hudson contains nearly 88 per cent of the county's population.

Each library system is "an association of autonomous local libraries working together to improve library service for all residents of a specified county or multi-county area."[9] Autonomy is a very important feature of New York State's public library system. Control by the State over the individual systems is minimal, and control by the system over the separate local public libraries is likewise weak. Historically this feature of New York State's system has probably been responsible for the widespread acceptance of system membership on the part of local public libraries.

The State does have two major ways to exercise control over library systems. First, in order to qualify for State aid the system must meet certain minimum standards of size,

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[9]University of the State of New York, The State Education Department, A Primer of Public Library Systems in New York State, Revised edition, The State Education Department, Albany, N. Y., p. 1.

staff, book acquisitions, facilities, etc. A system which failed to meet these standards might have its share of State aid withheld. Second, each system must submit an approved plan of library service to the Commissioner of Education prior to the receipt of State aid. Any subsequent revision of this plan needs the approval of the Commissioner. Entitlement to State aid can be withdrawn if the Commissioner revokes his approval of the system's plan of service (e.g., due to non-compliance with the provisions of the plan or the regulations of the Commissioner). Library systems also must adhere to other State requirements regarding the submission of annual reports, standard accounting procedures, and the certification of library professionals. At the current time no library system has failed to receive State aid due to failure to meet requirements or plans of service.

Library systems in New York engage in a wide variety of activities. Some of the most commonly offered services include:

(1) centralized book purchasing and processing

(2) consultant services to member libraries

(3) centralized storage of seldom used books

(4) delivery services

(5) bookmobile service

(6) rotating book collections

(7) workshops and training sessions.

The system, although financed with State aid, is not administratively controlled or directed by any State agency. It can freely choose to design and engage in those activities which might best meet local needs.

The governmental agency responsible for the administration of Federal and State aid to libraries is the Library Development Division (LDD). The division reports directly to the Assistant Commissioner for Libraries. In addition to supervising financial aid programs, it "maintains an extensive and varied program of planning, consultation, guidance, service, research and legal activity for libraries over the State. The Division has played a significant role in the growth of the systems structure."[10] Since State aid is allocated on the basis of a formula, the LDD is primarily concerned with compliance with objectives and proper administration of funds. The LDD does play an important role in the establishment of State-level

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[10]New York (State), Division of Evaluation, Emerging Library Systems, p. 143.

objectives and in the coordination of many diverse State-level library problems (such as the interlibrary loan system and serving the needs of the disadvantaged).

In this chapter we examined the State's role in library development. We noted that this role has been increasing over the past several years, and will likely continue to do so. State-aid formulas in selected States were presented and analyzed. Finally, New York State's system of public libraries was described with a view toward using this State as an example of how State-aid funds can be allocated in an optimal manner. The development of this allocation model will be presented in Chapter 4.

CHAPTER 3

STATE AID ALLOCATION MODEL

Introduction

In order to develop an efficient method for allocating State aid to public libraries, a primary task is to have an explicit statement of purpose or objectives of such State aid. The literature on State-aid objectives is evidence for the frequently-made criticism that most State library administrators (like many other decision makers), have only vague and high-sounding notions what their true objectives are. The published statements of objectives are often mere platitudes and as such are unsuitable for analytical forms of decision making. To cite a few examples:

(1) The basic objective in Pennsylvania is to "establish a system of district libraries throughout the Commonwealth so that all residents would have ready free access to a strong library."[1]

(2) "The State Library has the obligation to provide the people...with equal opportunity of library service."[2]

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[1]Ernst and Ernst, "Study of Library Services Program," unpublished report prepared for Pennsylvania State Library, Harrisburg, Pa., 1969, p. 6.

[2]Ibid., p. 13.

(3) "...to insure that every resident...has access to well developed library resources convenient to him." "...the objective of the State library is a better informed and culturally enriched citizenry."[3]

(4) "To stimulate local support for public library resources"[4] is one of the purposes of State aid, as well as

(5) "To equalize opportunity for residents of relatively poor areas..."[5]

It is one purpose of this research to formalize these vague and ambiguous statements of purpose into a set of objectives which must be explicit and consistent, and which are suitable for evaluating the degree to which they are achieved. Objectives should be explicit in order to channel State-level library planning and decision-making activities into those areas which contribute most heavily to the achievement of these objectives. Objectives should be consistent because conflicting objectives can operate at cross-purposes: pursuing one objective may automatically ensure

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[3]Commonwealth of Pennsylvania, Description and Justification of Program for Library Services of the State Library, Five Year Budget Summary for 1969-74, Schedule 6, p. 1.

[4]Ralph Blasingame, "A Critical Analysis of State-Aid Formulas," Library Trends, XIX (October, 1971), p. 252.

[5]Ibid., p. 252.

the non-fulfillment of another objective. If objectives are inconsistent, methods must be found to resolve their conflicting aims. One possible method for resolving conflicts is to impose constraints.

The need for suitable measures for evaluating the degree to which objectives have been achieved is crucial for rational planning and decision making. Not only are these measures important for assessing present performance, but they are also crucial for evaluating future alternative courses of action. They can also be useful for establishing standards of service against which other State library agencies with similar objectives can compare, and for developing an agency's own short-term goals.

State Library Objectives

Ideally, the objective of the State library administrator is to maximize the contribution of the State's libraries toward the fulfillment of society's goals. As noted in Chapter 1, the output of library service is the stimulated teacher or student, the scientific discovery, the informed voter, the successful businessman, etc. The library administrator at both the local and the State level should seek to maximize the value of these outputs. Unfor-

tunately this is an impossible task since these outputs are not measurable.

Since we cannot measure the ultimate effect of library service, we need to step backward to find a more suitable measure of library output. We can accomplish this by focusing on the interface between individuals and libraries. Within this framework, the output of library service is the exposure of individuals to documents of recorded human experience. This is the sine qua non of library activity, the essential purpose of a library's existence. It is assumed that library exposure measures serve as proxy variables for the unmeasurable concept of social utility. As library use increases, the assumption is that there is a greater level of achievement of ultimate societal benefits. Various possible measures of library use are discussed in the next section.

Library Performance Measures

Since the ultimate objectives of libraries are unmeasurable, numerous surrogate measures have been designed to gauge the degree of fulfillment of the library's objectives. These performance measures are often based on various concepts of document exposure, mentioned briefly in the pre-

vious section. The purpose of this section is to elaborate on these concepts and present several of the most prevalent performance measures.

The concept, document exposure, has been interpreted in many different ways in the literature. In its broadest context, document exposure can occur either directly or indirectly, in the library or outside the library, in the short run or in the long run.

Direct document exposure occurs when an individual applies at least one of his senses--seeing, hearing, or touching--to a document. For example, an individual reading a book is an obvious illustration of direct document exposure. Other examples are: observing a picture, watching and listening to a film, and touching Braille.

Indirect document exposure occurs when an intermediary transmits information to an individual as a substitute for direct document exposure. For example, the reference librarian communicating informational requests by telephone constitutes indirect document exposure; another example would be a story-telling session for children.

Document exposure does not always occur within the library itself. Photocopying documents for later use is

also document exposure, even if the original is returned. Similarly, documents are often circulated outside the library for periods of time ranging from a few days to a month or longer.

The concept of document exposure has a time dimension in addition to the aspects of time mentioned above. Not all library activity is designed to bring about immediate document exposure, although this objective is certainly a major consideration. Some library activities can best be classified as investment in future document exposure, similar in a business context to investment in capital assets. Illustrations of this aspect of document exposure are: the acquisition of a specialized book collection to support an anticipated future need where no need currently exists (particularly true in academic libraries); and campaigns to convert non-users to users.

A library administrator can formulate many useful objectives involving the concept of document exposures. One useful conceptualization of a library's objectives is to maximize document exposure defined in its broadest terms per dollar of input expenditures or cost. Alternatively, if a library administrator feels that his library's objec-

tives should be more narrowly defined (e.g., by neglecting long-run exposure), his library should seek to maximize this definition of document exposure per dollar of input expenditures.

The question can be asked, "Does the concept of document exposure encompass all the many benefits imparted by public libraries to society as a whole?" Libraries are frequently used by students as a study-hall and by the public at large for classes, for meetings of various kinds, for socializing, and for other functions unrelated to document exposure. In the strict sense, therefore, the concept of document exposure cannot possibly cover the myriad of other activities performed by libraries.

However, there is little doubt that the primary function of a library is concerned more with the delivery of document exposure than with the other functions mentioned above, and the measures of performance proposed in this research are based solely on the concept of document exposure. Partly, this exclusion of tangential benefits is a matter of expediency; partly it results from the notion that when a library is providing these additional services, it is not acting in its capacity as a library. If a library administrator wishes or needs to allocate resources

to provide these additional services, then he should perform the allocation process in two steps: a certain part of the budget is allocated to activities not related to document exposure, and the remainder is associated solely with activities dealing with document exposure.

Other performance measures have been proposed that are not based on document exposure concepts. It is felt that they are more useful for academic or special libraries than for public libraries. Typically these measures emphasize a reactive aspect of library service in which libraries perform well if they are able to respond to specific requests for documents or information. Examples of performance measures in this category are: the proportion of user demands satisfied,[6] document retrieval time,[7] and reader failure at the shelf.[8]

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[6]See Philip M. Morse, Library Effectiveness: A Systems Approach, MIT Press, Cambridge, Mass., 1968, and M. K. Buckland et al., Systems Analysis of a University Library, University of Lancaster Library, Lancaster, England, 1970.

[7]See R. H. Orr et al., "Development of Methodologic Tools for Planning and Managing Library Services: II. Measuring a Library's Capability for Providing Documents," Bulletin of the Medical Library Association, 56: 241-267 (July, 1968).

[8]John A. Urquhart and J. L. Schofield, "Measuring Readers' Failure at the Shelf," Journal of Documentation, 27: 273-86 (Dec. 1971), and 28: 233-41 (Sept. 1972).

Measuring Exposure

Three variants of document exposure measures have been suggested: exposure counts, item use-days, and document exposure times.

The first measure, exposure counts, is the crudest of the three measures. Each type of library use--circulation, renewal, browsing, interlibrary loan, and indirect use--is counted as one unit of exposure. Some of these types of library use typically are recorded in the routine operations of a library; others may be collected by sampling users, questionnaires, or direct observation by library personnel. All types of exposures may be added together, or else separate counts can be maintained for each exposure type.

This performance measure has several critical defects. If exposure counts are maintained separately for each exposure type, a multidimensional measure of library activity will be produced. There are two ways of treating this multidimensional measure. If different exposure types are not aggregated, the evaluation and allocation process can become quite awkward. Library performance may seem to be good for certain exposure types but not for other types.

What can be said about overall performance? Not very much. Similarly, resource allocation within a library would have to consider not only the direct effect of a particular allocation on each exposure type but also the interactions among exposure types.

On the other hand, if we combine all exposure types in some fashion (for example, by assigning weights to each exposure type and then aggregating), we may be adding essentially dissimilar things. For instance, how many browsed books are equivalent to one book which is circulated for two weeks? How many fast informational requests are equivalent to a reserve book used in the library for one hour? Complex issues of this sort need to be resolved in order to construct a valid performance measure based on aggregating all types of exposure counts. Another problem with this performance measure is that it is highly sensitive to the library's loan policy. A very short loan period requiring frequent renewals will tend to increase the total number of exposure counts for a library, while the actual use of the library and its materials could remain unchanged.

The item use-day is a somewhat more sophisticated per-

formance measure.[9] This measure counts each day or fraction of a day that a document is used as one item use-day. Thus, a book which circulates and is used every day for two weeks is equal to fourteen item use-days.

The advantage of this performance measure over the previous measure lies in the common unit of measurement for all exposure types. Each type of exposure, whether it be browsing, an informational request, or in-library use, counts as one item use-day, and at the end of each day these counts can be aggregated to find the total number of items used during that particular day. However, since circulated books may not be used every day of the loan period, an estimate is necessary of the average number of days of use per circulation. Hypothetically, the relationship between the average item use-day per circulation and the library's loan period is depicted in Figure 3.1.

The item use-day measure can be criticized for possessing some of the same arbitrary characteristics as the exposure count measure. The basic fault is an inherent "lumpiness" in its application: whether a document is

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[9] R. L. Meier, "Efficiency Criteria for the Operation of Large Libraries," Library Quarterly, 31: 215-234 (July 1961).

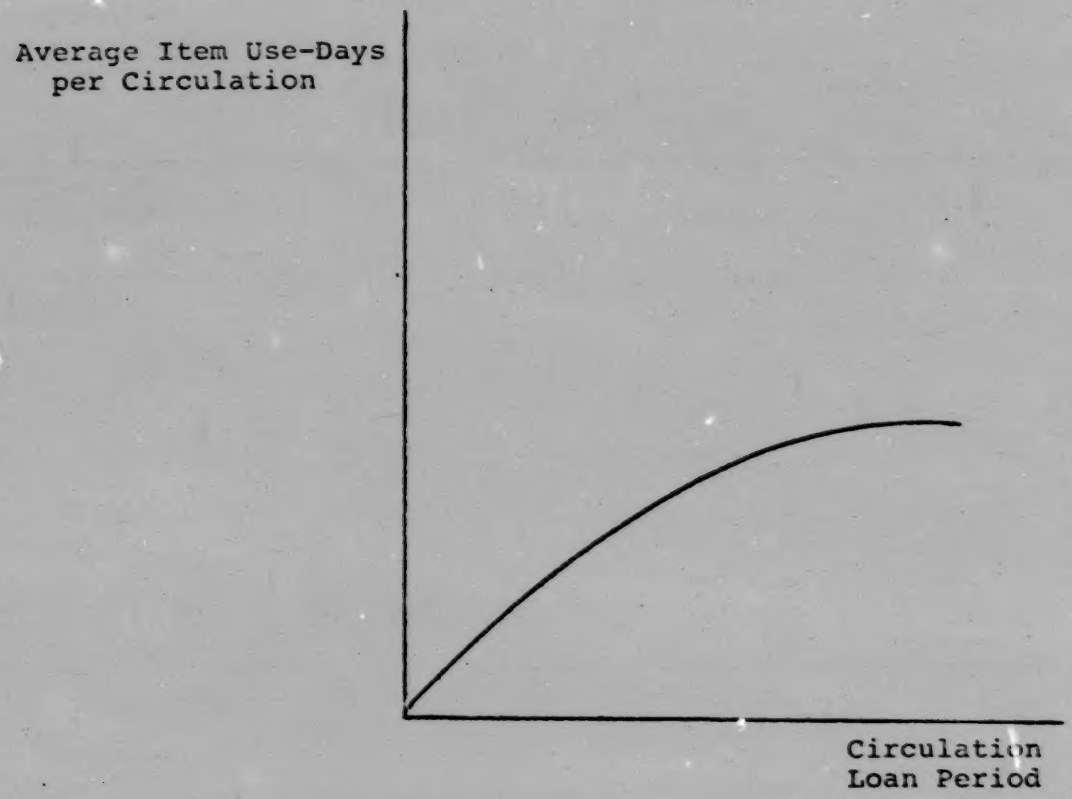

Figure 3.1: Average Item Use-Day as a Function of Circulation Loan Period

used for ten minutes or ten hours, each use results in one item use-day. Also, five one-hour exposures on five different days counts as five item use-days, while one five-hour exposure on one day only counts as one item use-day.

The third measure of a library's overall performance, document exposure time, is the most sophisticated of the measures. It is the most difficult to construct, but it does not possess the undesirable qualities of the other two measures. Document exposure time is a measure of the length of time that an individual has devoted to the library's document and information resources. The common unit of measurement for all types of library use is exposure time. Estimates of average exposure times for each type of library use are needed to construct the performance measure.

All three of the performance measures described above --exposure counts, item use-day, and document exposure time--can be criticized on the grounds that:

(1) The social benefits of library use are not the same for all documents. Some books may be "worthless," and others "valuable," in terms of the benefits which accrue to society by their use.

(2) For any particular document, there are differences in social benefits caused by different individual users. A chemist reading a chemistry book provides a benefit to society which differs from a high school student reading the same chemistry book.

These criticisms can be rebutted in several ways. If libraries acquire only documents which have social value, then they could not promote "worthless" library materials. On the other hand, if all documents have some redeeming value to society, then all document uses enhance individual and societal welfare. The second criticism is more difficult to parry. On the average, the amount of societal benefit probably does not differ greatly between individuals, so this criticism may not be of much practical importance. However, if it is felt that adjustments need to be made, an acceptable procedure would be to define population and document groups and apply different weights to particular population-document exposure subsets. The weights would be based on the relative benefit imparted to society by members of a population subset being exposed to a particular document subset.

In this section we discussed the development of performance measures for libraries. The choice of an adequate performance measure rests to a large extent in the hands of the library manager or, at the State level, in the hands of administrators of public library programs. In the next section we discuss the manner in which a suitable objective function for State level administrators can be formulated.

The Objective Function

For purposes of illustration, we shall examine the organization and development of New York State's system of public libraries. The agency within the State Government which deals with public libraries is the Library Development Division (LDD). This division, described more fully in Chapter 2, has three major decision variables under its control:

(1) State aid

(2) leadership effort

(3) consultative services.

State aid is the most important variable for purposes of this research. It will be assumed for purposes of this study that: the amount of leadership effort and consultative services offered to library systems remains constant;

that funds for these endeavors have already been allocated; that the present locational pattern of library units is stable; and that there is no substantial change in the quality of facilities or the quality and effort of library personnel. We wish to construct within this framework an analytical model for allocating State aid to library systems in order to maximize an over-all measure of library use.

Various possible measures of library use were discussed in the preceding section. Associated with each measure is a relative cost of collection. For example, the measure of exposure counts relies upon recorded exposures to a large extent and is therefore relatively inexpensive to obtain. On the other hand, document exposure time requires sampling users to obtain estimates of average exposure time for different exposure types - a procedure which would be quite expensive on a statewide basis. Cost considerations have had a strong bearing on choosing an appropriate measure for this research. While recognizing the limitations of the particular measure chosen, we assume that the basic objective in the allocation of State aid is to maximize the total number of circulations.

This variable has several useful characteristics. A circulation can be clearly and unambiguously defined as the recorded distribution of documents within the library to patrons. It includes the distribution of reserve documents if these transactions are recorded, but it normally does not include browsing or in-library use. Reference questions are likewise excluded. Circulation counts are collected by all public libraries in New York as a by-product of day-to-day operations. Furthermore, annual circulation counts are reported to the State as part of each library's yearly statistics.

The limitations of this measure of library use have been discussed in the preceding section. A major conceptual fault is that several types of library use are excluded, namely browsing, reference questions, or any unrecorded document use. Evidence within The Free Library of Philadelphia suggests that circulation accounts for nearly 90% of total library use (measured by document exposure time) and evidence gathered by Fussler and Simon indicates that for the large majority of a library's collection "nonrecorded use is roughly proportional to recorded use."[10]

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[10]Herman H. Fussler and Julian L. Simon, Patterns in the Use of Books in Large Research Libraries, The University of Chicago Press, Chicago, 1969, p. 115.

Hence, this fault is not so serious as it may seem on the surface. Another limitation is that the measure can be affected by a change in the library's loan policy. A library could increase the total number of circulations by shortening its loan period, thereby necessitating more frequent renewals. However, it is not clear that this method of increasing a library's circulation is foolproof. Frequent renewals are inconvenient for most patrons and a policy which encourages a short loan period might restrain or even repel some patrons.

No measure of library use is perfect; some are better than others, some more costly than others. The particular measure we have chosen is no exception. Its major advantage is that it reflects a library's over-all performance. A library which has adequate document resources and encourages their use, which monitors the needs and aspirations of its clientele, and which reacts to changes in its environment will be effective, and this will be evidenced by its circulation count. A library which has more than adequate resources but does not know its clientele will not be effective, and its circulation rate will be correspondingly low. A useful extension of this research is to de-

velop and implement a better over-all measure of library use.

Equity Considerations

If State aid were allocated strictly on the basis of maximizing total library use, most of the funds would probably go into library systems which served educated and relatively wealthy clientele. Disadvantaged groups, particularly those with less than a high school education and those living in sparsely populated areas, would probably receive very little State aid. For a variety of reasons State aid typically has been allocated in part by equalization factors. State administrators recognize that local resources are not equally distributed throughout the State, and their policies have emphasized the role of State aid in providing for equality of opportunity for library service."[11] The ALA's Standards for Library Functions at the State Level clearly establishes the principle: State financial aid for libraries should equalize resources and services across the state by providing extra help for those least able to finance sound services and facilities." Fur-

[11] See, e.g., Ralph Blasingame, "A Critical Analysis of State-aid Formulas," op. cit., p. 252 and Lowell Martin, Library Service in Pennsylvania: Present and Proposed, Pennsylvania State Library, Harrisburg, Pa., 1958, p. 128.

thermore, "every community should be expected to have at least a minimum level of public library service."[12]

There are two related concepts in these principles espoused by the ALA, and without delving too deeply into semantic problems, we need to distinguish between them. One concept, "equality of opportunity," is interpreted in this research to mean that public library services should be provided to all regardless of the available financial resources. Equalization factors in State-aid formulas which raise the level of funding in poor localities represent attempts to embody this principle. Thus, as an objective, "equality of Opportunity" means adjusting for differences in local resources. The second concept is "equality of use." While similar facilities may be available to all, this does not imply that these facilities will be utilized to an equal extent by all segments of the population. If library administrators wanted to equalize a measure of library use (e.g., number of circulations per capita) for all library systems, then clearly equalization factors based on local resources are inappropriate. The concept, "equality of use," addresses the problem of dif-

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[12]New York (State), Division of Evaluation, Emerging Library Systems, p. 239.

ferential needs for library service. This complex topic will be discussed in more detail in the next section.

We shall now discuss methods of incorporating an equity constraint into our allocation model. We view the constraint as consisting of two parts: library system minimum needs and library system economic resources.

Need for Library Service

Let us define m as the cost per capita of providing a minimum acceptable level of library service for a library system. Ideally we would like to define this minimum acceptable level of service in terms of units of output. Having established these output norms, State administrators could determine the proper mix of library inputs which would produce this level of output. It would then be necessary to establish the cost per capita of providing these inputs. Symbolically, if there are k types of library inputs (e.g., professional staff, reference books, and adult non-fiction books), let q_{ik} be the quantity of the k^{th} input necessary to provide the minimum acceptable level of output for library system i. Let p_{ik} be the average price of the k^{th} input for system i. Then the total cost to system i of providing the minimum acceptable

level of library service is: $mP_i = \sum_k p_{ik} q_{ik}$, where P_i is the population attributed to system i.

The cost per capita is given by:

$$m = \frac{1}{P_i} \sum_k p_{ik} q_{ik}.$$

The difficulty with this approach is that a library "production function" is required to establish the levels of inputs utilized for each level of output. Library inputs may be substitutable to a certain extent. Library staff and promotional effort, for example, might be substituted for document resources to produce the same level of output, and vice versa. If this approach is followed, one would seek the least cost solution to the quantity of inputs necessary to produce the minimum acceptable output level.

This "production function" approach may offer important insights into the various trade-offs which are possible in the provision of library service. Unfortunately, the mechanisms involved in the learning process and in the communication of information via alternative media channels are not understood very well. This lack of knowledge is not alleviated by the difficulty of even defining a "unit of output" of library service. For these reasons, a pro-

duction function approach will not be used in this research.

An alternative method for determining the value for m, and the method followed in this research, is to treat m as a policy variable. The value for m will be fixed by the decision makers at the State level. They might use a panel of experts for advice, but the value chosen will remain somewhat arbitrary. It will depend on the prevailing view of what constitutes a minimum acceptable level of service and many other value judgments. The sensitivity of the model to changes in the value of m will be studied in Chapter 4.

Local Resources

Given the cost of providing a minimum level of library service for each library system, who will supply the requisite revenues? The two most important sources of funds are the State and the local taxing authority. The amount of State aid allocated to the i^{th} library system will be designated S_i, and this will be determined from the allocation model. The amount of local revenues raised to support library service is not subject to the control of the State library administrators, and it will be determined exogenously.

Let Y_i be the potential local economic resources available to support library service in system i. Since S_i will depend on Y_i, we have defined Y_i in terms of potential resources for two basic reasons. First, it removes an element of gamesmanship from local-State relations. If State aid would rise when actual local tax revenues decreased, then State aid could be used to relieve the local tax burden. By choosing to define local resources as potential library support, we have taken this factor out of consideration. Secondly, this formulation does not discourage greater tax support for the local library system. There is no direct incentive for increased levels of support, but there are indirect incentives. Greater support will lead to improved document resources and increased library use, which will have the effect in our model of increased levels of State aid.

How do we measure Y_i? Potential economic support of library service depends on the underlying tax bases of each library system. Let there be j tax bases (gross sales, property values, income, etc.) and let B_{ij} be the j^{th} tax base in library system i. Define τ_j as the standard tax rate applied to the j^{th} tax base. The value of τ_j is not

the actual rate applied to the j^{th} tax base in any system. Rather it is a policy variable applicable to all systems in order to measure their ability to support library service. Then the potential economic resources available to support library service in the i^{th} library system is given by: $Y_i = \sum_j \tau_j B_{ij}$. The value of τ_j can either be subjectively determined by the decision-maker at the State level, or else an estimate of τ_j can be found from the frequency distribution of actual tax rates prevailing at any point in time. This latter method is to be preferred because it is less arbitrary. Several methods of estimating τ_j from actual data can be explored: median, mean, or mode. We will present a methodology for determining the τ_j in a subsequent section.

At the present time, the traditional tax base which is utilized to support library systems is the property tax. This is the same tax base which traditionally supports public education. In this latter arena, recent court cases have cast serious doubt on the constitutional issues related to the use of local taxable wealth as a determinant in the fiscal support of local school systems. In Serrano

v. Priest,[13] the Supreme Court of California decided on August 31, 1971, that the State of California's "school finance system is invalid because it classifies educational opportunity in the public schools on the basis of wealth."[14] On the other hand, in San Antonio Independent School District v. Rodriguez,[15] the U. S. Supreme Court by a vote of five to four decided on March 21, 1973 that the school system of Texas, which in many respects is similar to that of California, does not "discriminate against any definable class of 'poor' people...and, insofar as the financing system disadvantages those who, disregarding their individual income characteristics, reside in comparatively poor school districts, the resulting class cannot be said to be suspect."[16]

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[13] Serrano et al. v. Priest et al., 5 Cal. 3d 584, 487 P. 2d 1241.

[14] Federation of Tax Administrators, "School Finance, the Property Tax and the Courts," Chicago, March 15, 1972, (Mimeographed), p. 1.

[15] San Antonio Independent School District et al. v. Rodriguez et al., No. 71-1332, March 21, 1973.

[16] Ibid., pp. I-II.

The Rodriguez decision has calmed the crisis-like atmosphere which began with the Serrano case. However, the closeness of the decision and the views expressed in the minority opinions both portend a continuation of the constitutional battles. Since the landmark Serrano decision still may have broad implications for library financial arrangements, some of the pertinent details of this case will be presented. The complaint, filed by residents of Baldwin Park Unified School District, alleges that

> "As a direct result of the financing scheme (of the State of California)... The educational opportunities made available to children attending public schools in the Districts...are substantially inferior to the educational opportunities made available to children attending public schools in many other districts of the State."[17]

The complaint further alleges among other things that the current financing scheme:

> (1) "Makes the quality of education...a function of the wealth of the children's parents and neighbors, as measured by the tax base of the school district...
>
> (2) "Makes the quality of education...a function of the geographical accident of the school district in which (they) reside..."

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[17] Serrano v. Priest, p. 590.

(3) "Fails to provide children of substantially equal age, aptitude, motivation, and ability with substantially equal educational resources..."[18]

In reaching its decision, the court found compelling evidence of inequities in the State's school system finances. The evidence showed substantial differences in the amount of assessed valuation per pupil and in the level of expenditures per pupil among the school districts in the State. Furthermore, it found that local effort as measured by tax rates per $100 of assessed valuation differed widely among school districts, and indeed the court found the following contradiction to be true: districts which taxed themselves more heavily often ended up with lower per pupil expenditure levels due to a low tax base. As the court noted:

> ...so long as the assessed valuation within a district's boundaries is a major determinant of how much it can spend for its schools, only a district with a large tax base will be truly able to decide how much it really cares about education. The poor district cannot freely choose to tax itself into an excellence which its tax rolls cannot provide. Far from being necessary to promote local fiscal choice, the present financing system actually deprives the less wealthy districts of that option.[19]

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[18] Ibid., n. 1, p. 590.

[19] Ibid., p. 611.

In finding for the plaintiffs, the court did not invalidate nor declare unconstitutional the property tax per se. Its ruling affected only the current method by which State and local regulations and fiscal arrangements violated the equal protection clause of the Fourteenth Amendment to the Federal Constitution, which the court had previously ruled to be "substantially the equivalent" of Article I, sections 11 and 21 of the California Constitution. In fact, the court subsequently modified its opinion in Serrano on October 21, 1972, "emphasizing that the original decision did not constitute a final judgment on the merits and that the existing system of school financing is to remain in effect until it has been found to be unconstitutional and replaced by an appropriate new system."[20]

In summary, the Serrano case did not "question the validity of the property tax as an element, or even a major source, of revenue for school finance. The constitutional objection stems rather from the manner in which district property tax capacity is taken into account in determining the amount available for expenditure per pupil."[21]

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[20] 40 U. S. Law Week, p. 2339.

[21] Federation of Tax Administrators, op. cit., p. 1.

The constitutional issues raised by Serrano were not fully quieted by the Rodriguez decision. Many more court cases are likely to follow that will clarify which public services, such as libraries and education, involve "fundamental" interests and whether these interests are somehow abridged by inequitable State-local financial arrangements.

The legal, political, and economic questions which have been raised by Serrano will not be answered today, nor even within the next few years. There are many aspects of the current problem which need further probing and analysis. Part of this further work will involve broad-gauge measures of fiscal capacity and effort. No State financing scheme for schools or libraries has been scrapped yet, although there is a current, on-going search for a more equitable system. The property tax in some form or other will probably continue to be a cornerstone in any future arrangement.

Ability to Pay

A model for allocating State aid to multi-county systems of public libraries which depends on that area's financial capacity to raise tax revenues is sensitive to the method of measuring fiscal capacity. Local governments

can use alternative means of raising revenue so that one cannot merely look at only one tax, as for example, the property tax, and use this tax alone to estimate fiscal capacity.

Measures of ability to pay have been proposed in the literature. At the present time, the most complete study was conducted by the U. S. Advisory Commission on Intergovernmental Relations (ACIR) entitled Measuring the Fiscal Capacity and Effort of State and Local Areas (1971). The basic approach proposed in the ACIR study will be presented, and then modified and extended to meet the requirements of this research.

The basic purpose of the ACIR study is to construct an over-all index and measure of fiscal capacity. Factors included in the measure are not only the various sources of tax revenue, but also non-tax revenue and debt capacity. By far the most important element is tax revenue. In the construction of the aggregate measures, the ACIR recommends "a representative tax system" approach. The methodology of this approach is:

1. determine what tax bases are utilized.
2. review current practices with respect to each type of tax to ascertain its pre-

dominant or "representative" form.

3. locate tax base data or proxies for them.
4. obtain average rate or average yield for each tax by dividing total statewide tax revenue by total statewide tax base (or proxy for tax base).
5. calculate potential yield of each tax class by applying the average rate to the tax base for each area.
6. sum the potential yields calculated above to arrive at total tax capacity.

In this way, "the revenue capacity of any particular area is defined as the total amount of revenue that would result by applying, within the area, the national average rate of each of the numerous kinds of State-local revenue sources."[22]

The methodology of the ACIR study is quite useful, but it does not go far enough. What is necessary for this research is a method of calculating fiscal capacity to support a particular type of governmental expenditure, namely

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[22]U. S. Advisory Commission on Intergovernmental Relations, Measuring the Fiscal Capacity and Effort of State and Local Areas, Washington, D. C., Government Printing Office, 1971, p. 7.

library service. The question can be asked, "How can a standard rate of expenditure or level of support be established that local governments should apply toward the support of public libraries?" This question cuts across all the problems of equalization of resources, differential needs for library service, local fiscal autonomy, and local control of public libraries. Any proposed standard rate will be met by criticism: those local areas below the standard rate might argue that a higher rate would be an unnecessary tax burden; those local areas above the rate might counter that they should be rewarded for their herculean efforts.

The method proposed in this research involves an "average expenditure rate" for library service. This method has one very important advantage over an arbitrary method. The average expenditure rate, which we shall designate r, has the property that if it were applied to all local government budgets in the entire State, and these estimated amounts were aggregated, then this total <u>estimated</u> amount spent for libraries would equal the total <u>actual</u> expenditures for libraries. This property is desirable because it is economically neutral: it implies nothing about how much money <u>should</u> be allocated by local

governments to public libraries. Essentially it reallocates the total amount spent for libraries on an average-expenditure-rate basis.

This property can be demonstrated very easily. The average expenditure rate for libraries, r, is defined as:

$$r = \frac{\text{total local govt. expenditures for libraries in entire State}}{\text{total local govt. expenditures}}$$

$$= \left(\sum_i \pi_i E_i\right) / \left(\sum_i E_i\right)$$

where

E_i = total expenditures of the i^{th} governmental unit

π_i = proportion of the budget of the i^{th} governmental unit allocated to libraries

Multiplying both sides of the equation by $\sum_i E_i$, we have,

$$\sum_i r E_i = \sum_i \pi_i E_i$$

Thus the sum of the potential yields using the statewide average expenditure rate is equal to the total actual expenditures for libraries by local governments.

We do not necessarily accept as proper the current levels of funding for libraries, either on the part of local or State government. The question of the optimal allocation of funds for libraries within the total local

and State budget remains unanswered. An average financing system merely uses the current level of funding as a starting point and computes the revenue which would be generated if all jurisdictions adopted the "average" tax rate. In the absence of a normative model for determining the optimal level of funding by local and State governments, this method is also desirable for mathematical tractability, as demonstrated above.

A critical limitation of the average financing method is the danger of suboptimization. If the current level of funding is not optimal, then the reallocation of resources will achieve a higher value on the social welfare function, but it will not be the highest level attainable. This latter level can only be achieved if the original total expenditures for libraries were optimal. A full study of this problem would have to focus on the optimal mix of support from all three levels of government, local, State, and Federal. At each level of government, the determination of an optimal budget for libraries involves a balancing of the relative costs and benefits of libraries vis-à-vis all other expenditure categories. Furthermore, the interaction between levels of government must be taken into

account. Even if this analysis may be infeasible at the present time, it represents a conceptual framework for improving the average financing system.

Summarizing the methodology and formulating it in mathematical terms, we will

(1) determine what tax bases are utilized at the local level and their representative form, which we will designate B_{ij} for the j^{th} tax base and the i^{th} governmental unit

(2) collect tax base data or proxies for such data

(3) obtain average tax rates for each tax class by the formula

$$t_j = \frac{\text{Statewide yield of tax class } j}{\text{Statewide base for tax class } j}$$

$$= \frac{\sum_i t_{ij} B_{ij}}{\sum_i B_{ij}}$$

where t_{ij} represents the tax rate applied by the i^{th} governmental unit to the j^{th} tax base.

(4) calculate average library expenditure rate by the formula,

$$r = \frac{\sum_i \pi_i E_i}{\sum_i E_i}$$

(5) Calculate the potential revenue yield, or the estimated fiscal capacity, for each tax class by multiplying the average tax rate t_j by the appropriate tax base measure for each local area,

$t_j B_{ij}$ = estimated fiscal capacity for tax class j in government unit i

(6) Aggregate these estimated fiscal capacity figures for all tax classes to arrive at an estimate of total fiscal capacity,

$\sum_j t_j B_{ij}$ for each government unit i.

(7) Multiply the total fiscal capacity for the i^{th} governmental unit by r to arrive at the "estimated capacity for library support,"

$r \sum_j t_j B_{ij}$ for each government unit i.

Recall that we have previously defined Y_i as the economic potential to support library service in the i^{th} library system. We can now define Y_i explicitly as:

$$Y_i = r \sum_j t_j B_{ij}$$

where B_{ij} is the j^{th} tax base for the i^{th} library system area

t_j is statewide average tax rate for the j^{th} tax class

and r is the statewide average expenditure rate for libraries.

If we sum Y_i over all of the i library system areas, we have

$$\sum_i Y_i = r \sum_i \sum_j t_j B_{ij}$$

If we assume that there is no debt financing and that revenues equal expenditures, then

$$\sum_i Y_i = r \sum_i E_i$$

$$= \frac{\sum_i \pi_i E_i}{\sum_i E_i} \quad \left(\sum_i E_i \right)$$

$$= \sum_i \pi_i E_i$$

Thus, the sum of the estimated capacity for library service is equal to aggregate actual library support.

The data needed to generate these estimates are:

(1) the amount of revenue collected for each tax class

(2) tax base data or suitable proxies

(3) library revenue from local government sources

(4) total local government revenue.

Equity Constraint

We have now defined for each library system minimum acceptable levels of service and the estimated capacity for library support. If potential local resources are not sufficient to cover these costs, then State aid will be necessary at least to fill in the gap. We recall that for library system i, S_i is defined as the amount of State aid and P_i is the population served by the system. In order to meet conditions of equity, the sum of local resources and State aid must not be less than the cost of providing the minimum levels of service. Thus, we can write:

$$S_i + Y_i \geq mP_i \quad \text{for all } i.$$

Substituting $r \sum_j t_j B_{ij}$ for Y_i, we have the constraint:

$$S_i + r \sum_j t_j B_{ij} \geq mP_i \quad \text{for all } i.$$

If $r \sum_j t_j B_{ij} \geq mP_i$, this constraint is automatically satisfied. This condition would most likely be found in relatively wealthy areas of the State where the potential tax revenues are quite high. This does not imply, however, that a wealthy area would not receive State aid. Wealth, education, and high intensity of library use are strongly correlated so that the objective of maximizing

total library exposure counts would normally imply greater State aid to areas of high wealth. The equity constraint insures that even poor districts receive some State aid. A budget constraint places an overall limit on the total amount of aid which can be distributed.

Determinants of Library Use

If State library administrators were satisfied that the mathematical model described earlier adequately depicts their allocation problem, and if all parameters could be accurately estimated, then the output of the model would be the amount of State aid to be distributed to each library system. This will be done for New York State in the next chapter. This result would be interesting and beneficial to some individuals and groups of individuals, but the model would have only limited applicability answering hypothetical questions or as part of a larger feedback control system. For example, questions such as the following might arise:

(1) What would occur if the State aid budget were increased (or reduced) by $1,000,000?

(2) If a particular library system became more effective utilizing State aid (e.g., by means

of a successful experimental reading program), how would this affect the optimal allocation of State aid, and what would be the effect on total library service in the State?

(3) What would occur if the prescribed minimal support level for library service were raised by $1.00 per capita?

(4) How can information about the current allocation of State aid lead to better performance in the next and subsequent periods?

The analysis of these hypothetical and normative questions is most important for State administrators as well as for librarians, budget officials, and legislators. The results of the allocation model are not sufficient to answer these questions. However, the results can be used as inputs to another model which predicts library use as a function of State aid and other variables, and then both models in tandem can be applied in the analysis of the type of hypothetical questions mentioned above. The formulation and estimation of the prediction model are discussed in this section.

We have previously stated that the objective which

State administrators seek to maximize is an over-all measure of library use. For purposes of this research the variable chosen to represent library use is number of exposure counts. Library use results from the interactions of many diverse forces, as a profusion of user studies has demonstrated. There is no single variable which determines library use, but there are numerous variables which are correlated with use. Since the purpose of our model is prediction, we will be choosing the set of variables which are the "best" predictors of use. Indeed one of the findings of this research is that some of the variables often identified in individual library user studies as explaining or being highly correlated with library use are not the best predictors of use for multi-county library systems.

Three categories of variables will be included in the prediction model: uncontrollable variables, variables subject to control at or below the library system level, and control or decision variables at the State level. A functional relationship between library use and these three categories of variables will be estimated, and the adequacy of a multiple regression prediction model will be explored.

Conceptually, we will represent the three categories of variables in the following way: Let

$\underset{\sim}{U}_i$ = vector of uncontrollable variables for library system i

$\underset{\sim}{Q}_i$ = vector of variables subject to control at or below the library system level for system i

S_i = amount of State aid received by library system i

C_i = number of exposure counts for library system i

Then the prediction model will involve an analysis of the functional form,

$$C_i = f\ (\underset{\sim}{U}_i,\ \underset{\sim}{Q}_i,\ S_i)$$

In the remainder of this section, the choice of variables included in the vectors $\underset{\sim}{U}_i$ and $\underset{\sim}{Q}_i$ will be discussed.

For the State library administrator, these prediction equations can be useful in several different ways. First, at the State level, variables in the first two categories can be considered as exogenous variables. Given values for these variables for any library system, predicted library use becomes solely a function of State aid. Any increase

or decrease in State aid will lead to a predicted change in the level of library use. Secondly, since the prediction equations will be estimated statistically, we can formulate probabilistic statements concerning the attainment of various possible levels of library use.

In the remainder of this chapter and in Chapter 4, the results of the empirical investigation of the prediction model and the allocation model are presented. The data for the models are described fully in Appendix A. Only New York State data were analyzed since this is one of the few states which regularly collects and publishes a large variety of library statistics, including information on library use.

Although the data pertain only to New York, the results can be generalized to other states. Historically, New York was the first major State to develop successfully a substantial State-aid program. Many other states, most notably California, Illinois, and Michigan, have patterned their library system structure and aid programs on the New York model. In summary form, the basic features of this model are: submission of a plan reorganizing public libraries into systems which are separate legal entities,

providing free access to all residents, establishing a central headquarters library, and supporting the system with local and State taxes.

In addition to the New York model, two other models can be identified. The Pennsylvania model, also followed by Massachusetts, New Jersey, and Rhode Island, features regional and district libraries. Designated by the State, these libraries form a hierarchy of library service. Local libraries receive aid if they agree to perform certain specified functions. Regional and district libraries may receive separate State funding, usually on a per capita basis (see Chapter 2 for details of Pennsylvania formula). The Maryland model, on the other hand, does not build a hierarchy of service. This model features a modified matching system for aid in which the State provides a fluctuating percentage of local library revenues.

We shall now report the major results of our investigation of variables to predict library use. A full description of the variables, data sources, and technical material to support our findings appears in Appendix A.

For a single library, most user studies typically find that two uncontrollable variables which are highly correla-

tive with library use are education level and proportion of school age children in the population served by that library. These variables reflect the well-documented phenomena that:

(1) public libraries typically attract and serve the more highly-educated people in the population at large, and

(2) approximately one half of all public library users are school-age children.

For multi-county systems of public libraries, however, differences between communities and population groups are more balanced; hence the two aforementioned variables are not predictive of library use at the library system level.

This result can be demonstrated statistically by means of simple linear regression. Defining the independent variable, L, as the proportion of school age children residing in the system's jurisdiction and the dependent variable, C, as the number of circulations in each library system, one would expect a positive relationship to be shown. In fact, however, a negative relationship is found. The estimated regression equation is shown in Table 3.1. The quantities enclosed in parenthesis be-

low the equation are standard errors of regression coefficients. The summary statistics for the equation are: coefficient of determination, adjusted for degrees of freedom, $\bar{R}^2 = .232$; the standard error of estimate, $S_e = 30.07$. We can test the significance of the coefficient for

Table 3.1

Estimated Regression of Number of Exposure Counts upon Proportion of School Age Children, New York State

$$C = \underset{(66.25)}{215.61} - \underset{(2.59)}{7.03}\, L$$

$n = 22$

$\bar{R}^2 = .232$

$S_e = 30.07$

Note: C measured in units of 100,000

L measured in percentage units

L at the .01 level by computing the t statistic, $t = \frac{7.03}{2.59} = 2.714$. Large values for t lead to the conclusion that the coefficient is significant. The critical value of t for 20 degrees of freedom is 2.845. Since the computed value is less than the critical value, we conclude that the coefficient for L is not significant.

Investigation of the relationship between library use and education revealed similar insignificant results at the level of multi-county library systems. Other uncontrollable variables were scrutinized for their value in predicting library use, and for purposes of this research, two variables were selected as the components of the vector of uncontrollable variables, $\underset{\sim}{U}_i$:

(1) Population density, D

(2) Total local government expenditures for libraries, G

There certainly are other uncontrollable variables correlated with library use in addition to these two variables. Several obvious omissions are income level and distance from nearest library unit. However, it is felt that in most cases these omitted variables are highly collinear with the included variables (income with total local government expenditures and distance with population density) so that the incremental effect of including these variables would be negligible. Furthermore, inclusion of these variables could quite possibly introduce a problem of multicollinearity, a problem which will be addressed directly below.

Population density, D, affects library use in a complex way. The population density of the area served by a library system is the ratio of the system's population to its area measured in square miles. Typically, as population increases library use will probably increase, but as the area served increases, library use is likely to decrease. The net effect of these counterbalancing forces for New York State's library system areas is portrayed graphically in Figure 3.2.

The relationship between population density and library use is clearly not linear. Several transformations of the data were examined to adjust for non-linearities. The functional form which provided the best linearization was the natural log of population density, Ln D. The results of the regression of C on Ln D are discussed in Appendix A and are shown graphically in Figure 3.3.

The second uncontrollable variable included in the prediction model, total local government expenditures for libraries in system i, will be denoted by G_i. The relationship between total local government expenditures for libraries and aggregate library use is very strong, as shown by Figure 3.4.

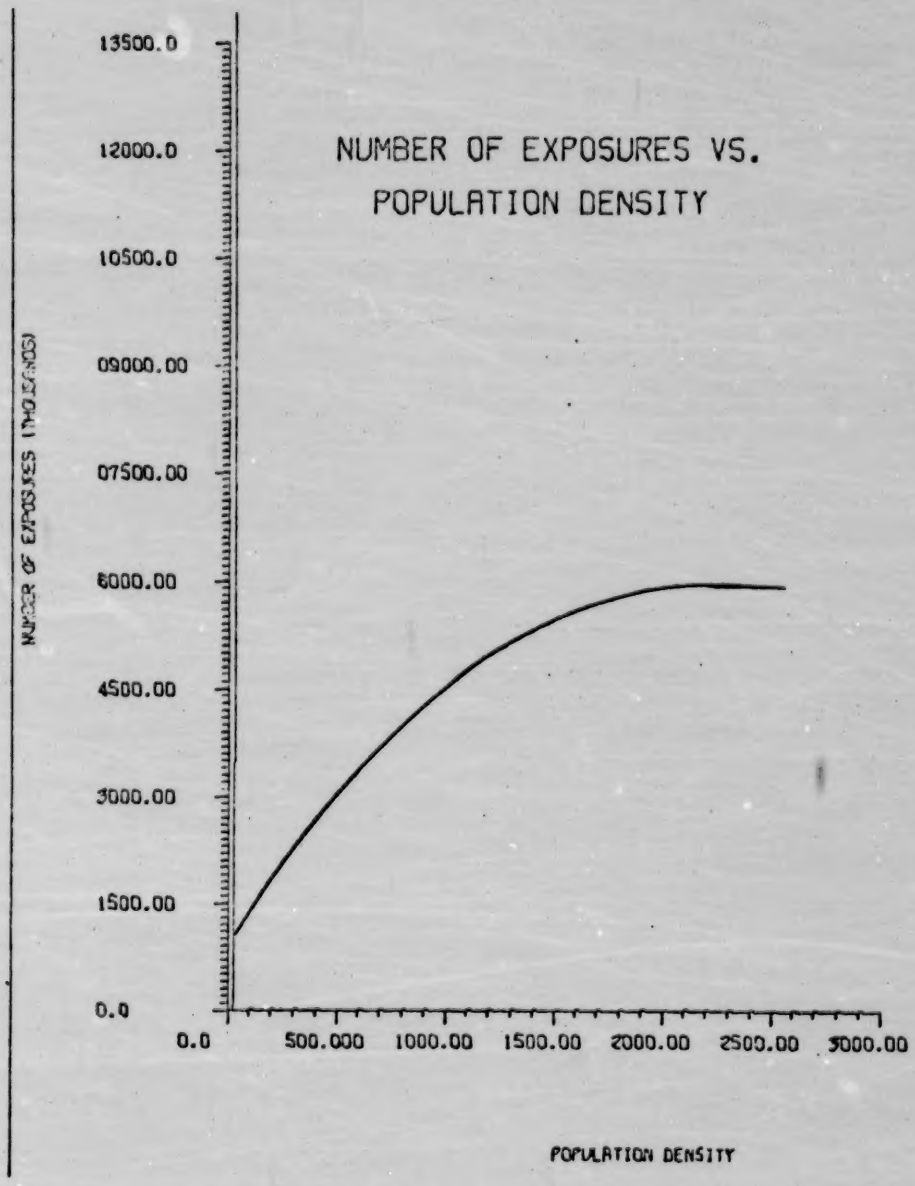

Figure 3.2: Relationship Between Library Use and Population Density, New York State

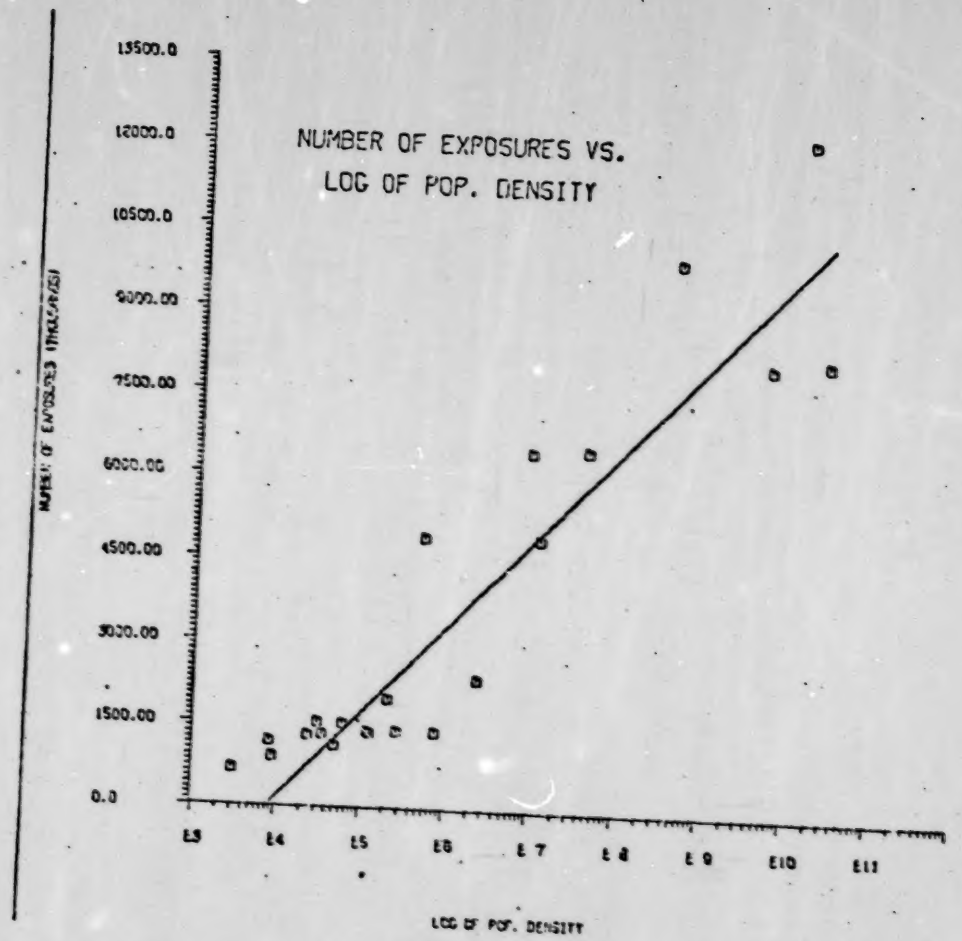

Figure 3.3: Relationship Between Library Use and the Natural Logarithm of Population Density, New York State

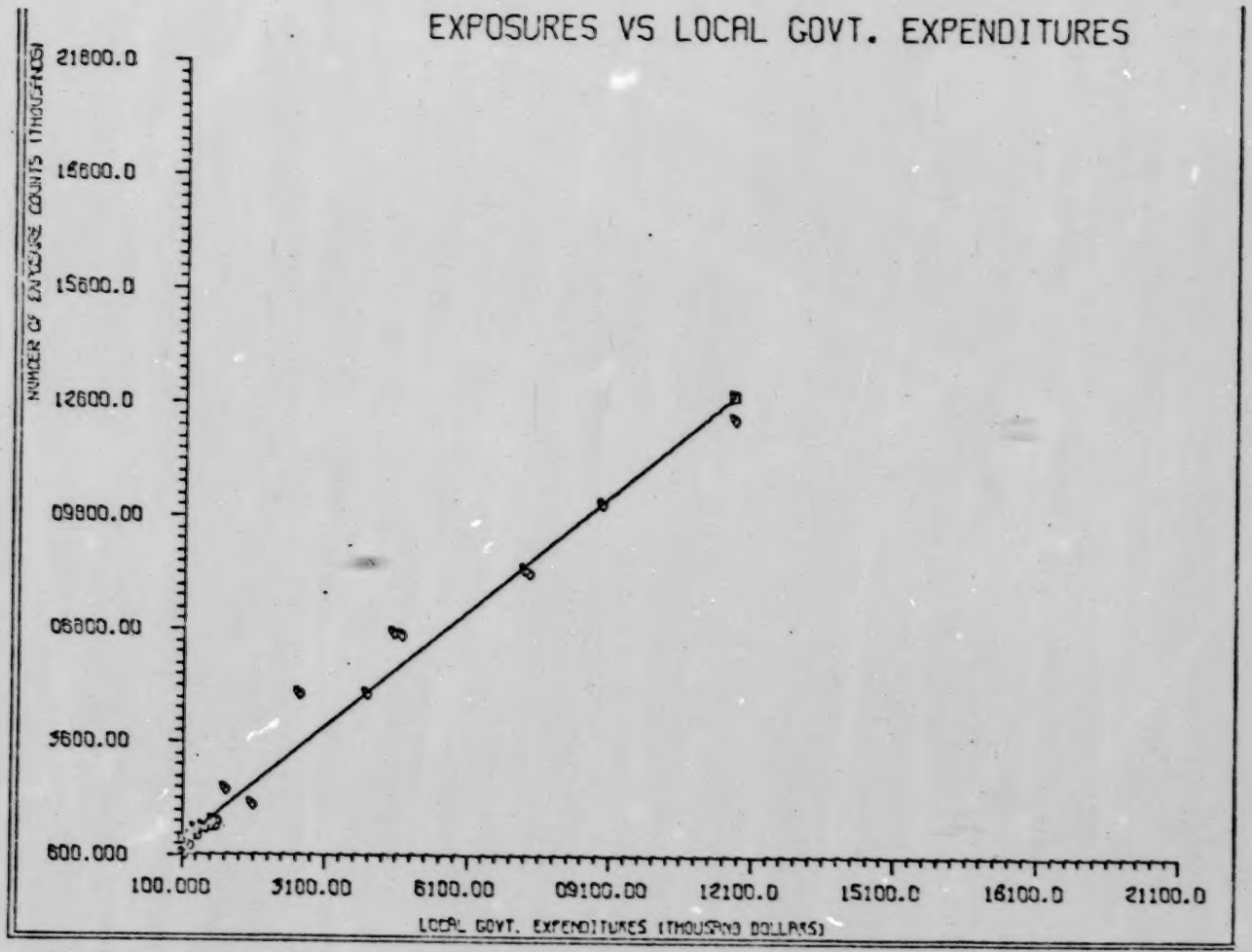

Figure 3.4: Relationship Between Library Use and Total Local Government Expenditures for Libraries, New York State

The regression results indicate that 97.8% of the variation in number of exposure counts is accounted for by the single variable, total local government expenditures for libraries. If only one variable were chosen to predict library use, then the variable G is the most accurate predictor of use among the variables studied. The hypothesis that the regression coefficient of G is equal to zero is very strongly rejected at the .01 significance level.

Another type of variable which will be included in the prediction model is the vector of variables subject to control at or below the library system level. We have denoted this vector $\underset{\sim}{Q}_i$. Four variables were examined for possible inclusion in the vector $\underset{\sim}{Q}_i$: number of books added by local purchase or gifts, K; total book holdings, H; expenditures for library materials, M; and total operating expenditures, O.

All four variables were found to be strongly related to number of exposure counts. The coefficients of determination, $\bar{R}^2$, indicate that between 79.8% and 93.0% of the variation in exposure counts can be accounted for by the simple regression of exposure counts on one of the four variables. The best single predictor of exposure counts

in this group is number of books added by local purchase or gifts, K. The regression results shown in Appendix A indicate that the regression of C on K has the highest coefficient of determination, $\bar{R}^2$, the smallest standard error of estimate, S_e, and the largest computed F-value. Furthermore, using the increment to R^2 as criterion, it was found that the addition of another variable does not significantly contribute to the predictive power of K alone. Thus the vector $\underset{\sim}{Q}_i$ will include only one variable, K, number of books added by local purchases and gifts.

The last factor which will be included in the prediction model is the amount of State aid received by each library system. This is the only variable subject to control by State-level administrators. The value of this variable for each library system can be determined by a distributional formula established by a State legislature, as it is done currently, or it can be determined in a normative way, as proposed in this thesis, by means of a mathematical programming allocation model.

The relationship between State aid, S, and number of exposure counts, C, is shown graphically in Figure 3.5. The relationship is fairly strong, but most of the observa-

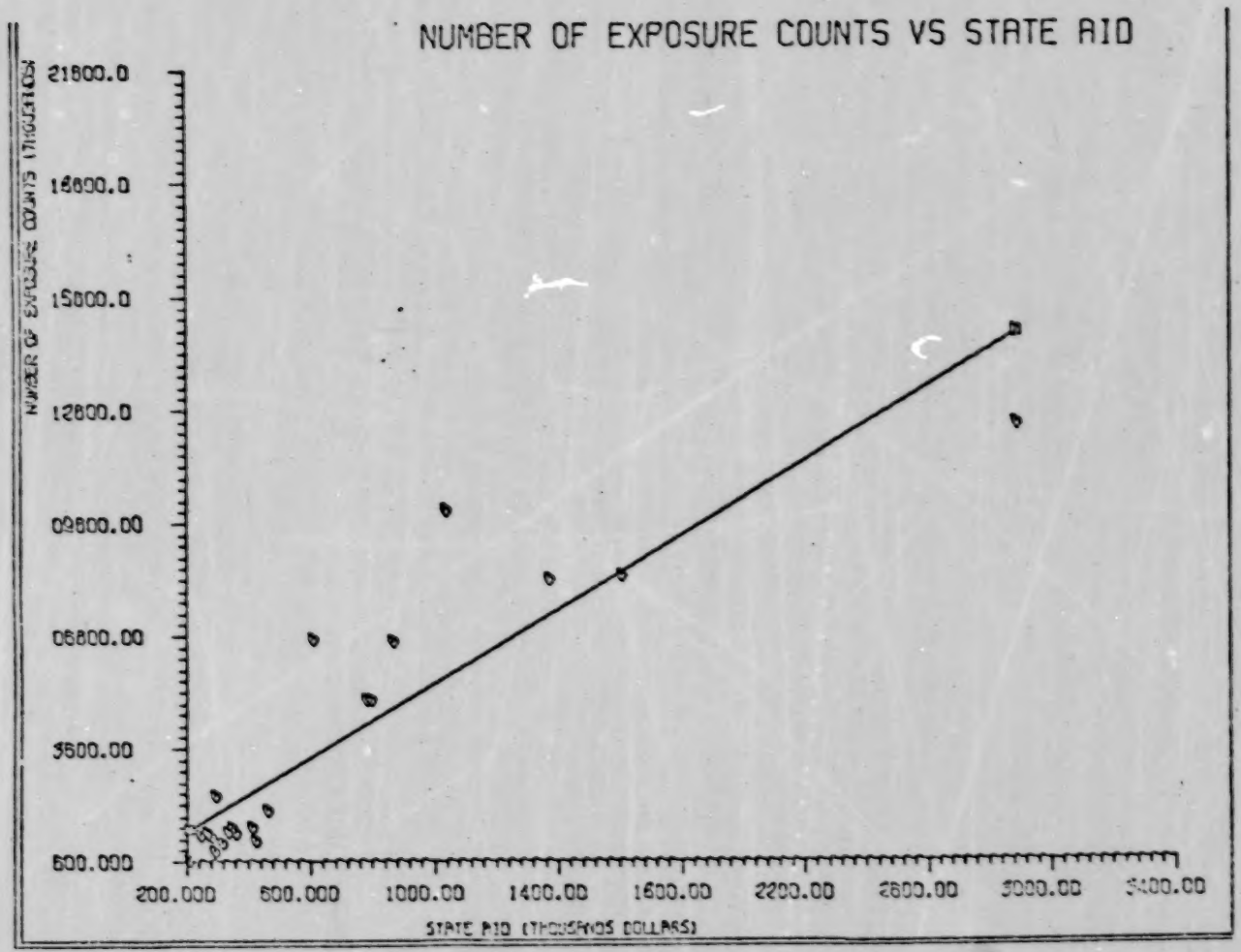

Figure 3.5: Relationship Between Library Use and State Aid, New York State

tions are clustered around the lower left corner of the graph. The eight observations lying outside this cluster will heavily influence the slope of the estimated regression line.

The results of the regression of C on S are presented in Appendix A. The results indicate that for State library administrators the single variable under their control, S, is a fairly accurate predictor of library use. This variable alone accounts for 79.7% of the variation in number of exposure counts. This is one of the most useful findings of this research as far as State library administrators are concerned.

We have now identified the variables which will be included in the prediction model. Clearly, only a few of the many possible variables have been specified. However, the tests performed thus far suggest that the model will provide reasonably reliable predicted values of library use. A detailed report of the entire prediction model follows.

Recall that we have identified these variables as predictors of library use:

Ln D = natural logarithm of population density for each library system

G = local government expenditures for libraries for each library system

K = number of books added by local purchase or gifts for each library system

S = amount of State aid allocated to each library system

We studied each variable individually as a predictor of library use. Now we need to study these variables together in order to judge their joint effectiveness in predicting library use.

A multiple regression of number of exposure counts on the four independent variables reveals a very strong linear relationship, as shown in Appendix A. The coefficient of multiple determination $\bar{R}^2$, adjusted for degrees of freedom, is equal to .977, and the F statistic for the significance of the over-all equation is equal to 226.70. Although these results are excellent indications of a very strong linear relationship, an examination of the regression coefficients and their standard errors suggests that several, if not all, of the independent variables are collinear.

For purposes of this research, the presence of multicollinearity has two serious consequences: there is a loss of precision in the estimation of the regression coeffi-

cients, and the estimation procedure is very sensitive to the particular set of sample observations. Multicollinearity makes it statistically impossible to estimate the relative influence of any of the variables; they are not mutually independent.

The detection of multicoilinearity in this model is straightforward. Presented in Appendix A, the evidence reveals relatively large values for standard errors of the coefficients (and therefore insignificant t values), very high pairwise correlation coefficients between all explanatory variables, and finally, rejection of all tests of the hypothesis that the explanatory variables are statistically independent of each other.

The search for a resolution of the multicollinearity problem can lead in many different directions. The technique utilized in this research is a respecification of the basic model. We define:

ΔG = annual rate of change of local government expenditures for libraries

ΔK = annual rate of change of number of books added by local purchase or gifts

The interpretation of the respecified prediction model now has an added dimension, one which includes the dynamic as-

pect of rates of change. We shall now examine the statistical properties of this new relationship:

$$C = f(\text{Ln } D, \ \Delta G, \ \Delta K, \ S).$$

A multiple regression of number of exposure counts, C, on these four independent variables indicates a strong linear relationship, as shown in Appendix A. The coefficient of multiple determination, adjusted for degrees of freedom, is equal to .976, and the F statistic for the equation is equal to 209.63. An examination and comparison of the standard errors of the regression coefficients reveals that the respecified model is not much of an improvement over the previous model. Furthermore, there is still evidence of multicollinearity, particularly between the variables Ln D and S. However, if Ln D is omitted from the model, the results indicate that the remaining variables S, ΔG, and ΔK are mutually independent.

The multiple regression results of number of exposure counts on these three variables is shown in Table 3.2 and Appendix A. These results compare favorably with the results of the previous models. The summary statistics are: $\overline{R}^2 = .962$, $S_e = 6.74$ and $F = 175.63$. The magnitude of the F value indicates that the equation is highly significant

Table 3.2

Multiple Regression of Number of Exposure Counts on Change in Local Government Expenditures for Libraries, Change in Number of Book Additions, and State Aid

Regression Equation and Results
$C = -3.4752 + 4.4629\ \Delta G - .3093\ \Delta K + .4948\ S$
$(2.2651)\ (0.5817)\quad (0.0624)\quad (0.0254)$
$\bar{R}^2 = .962$
$S_e = 6.74$
$F = 175.63$

at the .001 level.

An examination of the standard errors of the coefficients and their associated t values reveals the improvement in the model due to the elimination of the variable Ln D. With the exception of the constant term, every t value in the simpler model is greater than the corresponding value in the other model. The most noticeable change occurs in the precision with which the coefficient of S is estimated -- the t value increases from 8.75 to 19.49 by eliminating Ln D from the prediction model.

In conclusion, the best set of variables which state library administrators can use for predicting library use for multi-county systems of public libraries are:

(1) the rate of change of local government expenditure for libraries, ΔG,

(2) the rate of change of number of books added by local purchase and gift, ΔK, and

(3) State aid.

An alternative approach to the forumulation of the prediction problem was also considered, the results of which are reported below. If we define ΔC as the change in number of exposure counts for each library system

between 1967 and 1968, we can try to formulate a model to predict ΔC as a function of State aid and other variables. Using New York State data, the multiple regression results are not clearly indicative of the relative importance of State aid in determining the change in the number of exposure counts. Defining the dependent variables as:

ΔG = change in local government expenditures for libraries between 1967 and 1968

ΔK = change in number of books added by local purchases or gifts between 1967 and 1968

ΔS = change in State aid received by each library system between 1967 and 1968

P = population attributed to each library system, 1970 Census.

The multiple regression results are shown in Table 3.3. Although both equations indicate statistically significant relationships at the .01 level between ΔC and the explanatory variables, in neither equation is the coefficient of ΔS significant. This finding leaves unanswered the question of what is the precise relationship between changes in State aid and changes in the number of exposure counts.

These equivocal results in this formulation of the

Table 3.3

Regression Results for Prediction of ΔC

Regression Equation	$\bar{R}^2$	S_e	F-value
(1) $\Delta C = -.7117 + .5768\Delta G + .0582\Delta K - .0672\Delta S$ $\quad\quad\quad

prediction model support several alternative hypotheses. One possibility is that the activities for which increased levels of State aid are used are not likely to lead to significant changes in the level of exposure counts. This hypothesis assumes that the activities resulting in high levels of exposures are ordinarily engaged in, and changes in the level of State aid will be used, therefore, to support less rewarding projects.

Another possible explanation for the results is that the range of data may be too narrow to reveal the true relationship between ΔC and ΔS. For New York State data, ΔS ranges from a low of -$358,800 to a high of $348,600 while ΔC ranges from a low of -492,000 to a high of 611,000 exposure counts. Under this hypothesis a greater range of variation for ΔS would be necessary to gauge the true nature of the relationship.

A third possibility is that ΔC is not related to ΔS directly, but rather an underlying lag structure may exist such that ΔC is related to the previous year's value of ΔS. An even more complicated lagged relationship may also exist. This hypothesis asserts that a change in the level of State aid does not affect the current level of exposure

counts. Activities undertaken with increased levels of State aid may require a year or more to produce increased levels of exposure counts. For example, the aid represented by ΔS may be earmarked by some systems for experimental programs not intended to produce exposures in the current year.

More research is necessary on the development of a model to predict changes in the level of exposure counts. The results reported above are unclear and they could support several alternative hypotheses. The additional research should focus on how library systems allocate the year to year changes in the level of State aid; how a greater range of variation in ΔS affects ΔC; and if an underlying lag structure exists.

Summary of the Allocation Process

Figure 3.6 presents a flow chart of the allocation process and the estimation of total library use. The objective function in the mathematical formulation that we wish to maximize is the sum of $a_i S_i$ for all i library systems. The coefficient a_i expresses the relative importance in library system i of State aid in determining library use. It is measured in units of exposure counts

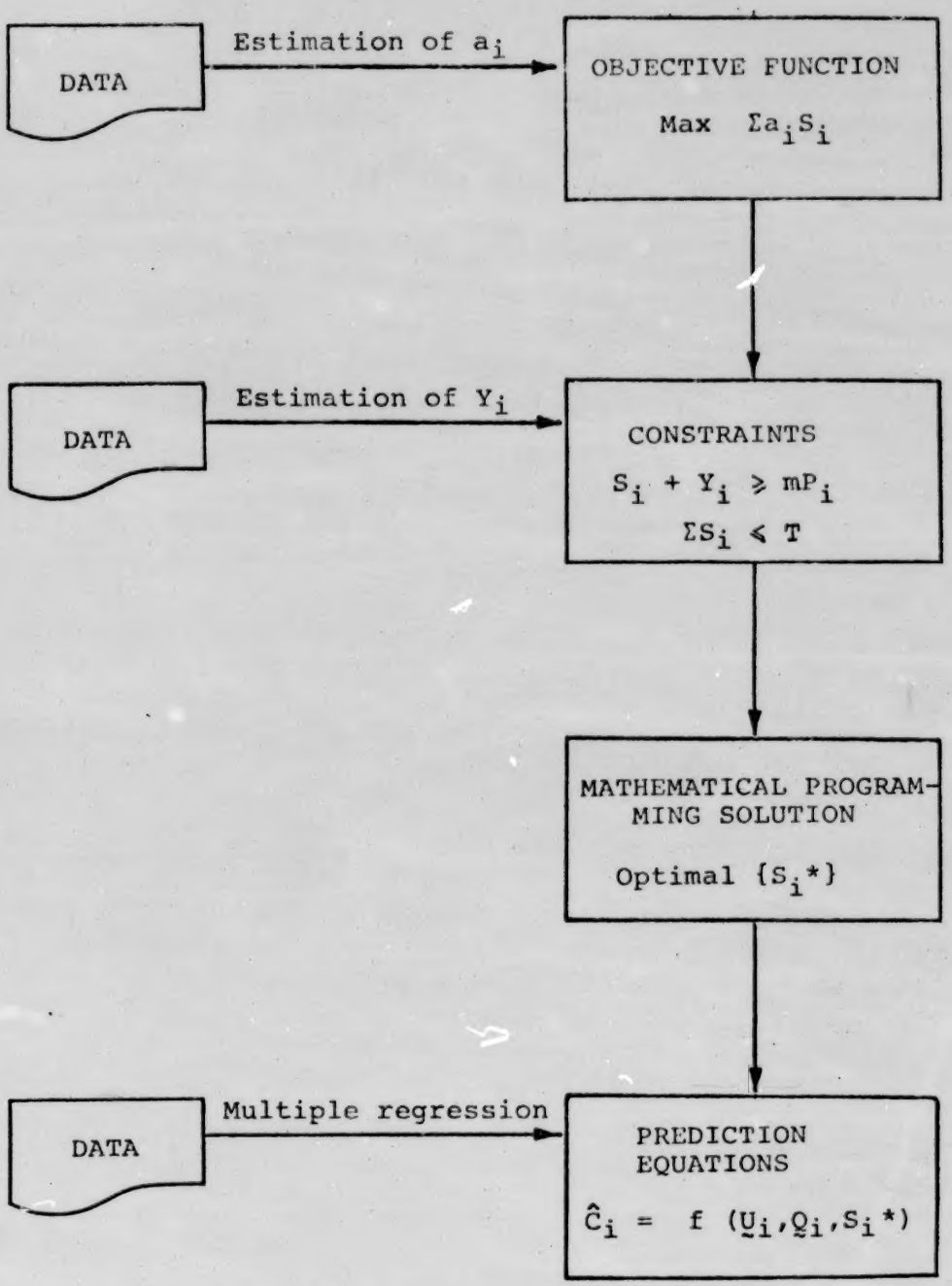

Figure 3.6: Flowchart of the Allocation Model

per dollar of State aid. If we assume that all systems are equally effective in using State aid for the provision of library service, then $a_i = 1$ for all library systems. Otherwise the coefficients a_i will differ. The determination of values for a_i will be discussed in the next chapter.

The constraints placed on the model specify that every library system will receive enough potential library revenue to meet minimum acceptable levels of service, where library revenue is the sum of State aid and potential local revenue. Furthermore, the sum of the amounts allocated to library systems as State aid cannot exceed the total budget for State aid.

The result of the mathematical programming formulation will be a set of values for each S_i which represents the optimal allocation of State aid, given the constraints on the system and objective function. These optimal values of S_i can be designated S_i^*. These values S_i^* can be substituted into the prediction model developed in this chapter to determine the expected level of library use, given this allocation of aid. This will be done for New York State in the next chapter. A summary of the vari-

ables used in the allocation and prediction models appears in the glossary.

Summary

In this chapter we presented the formulation of a model for the efficient allocation of State aid to public libraries. We developed the objective of the model and the set of constraints which limit or restrict the decision making process. Then we presented the formulation and results of the prediction model which forecasts library use as a function of uncontrollable variables, factors subject to control at or below the level of the library system, and State aid. Finally the general framework of the entire allocation process was described. The next chapter will describe the results of the mathematical programming portion of the allocation process.

CHAPTER 4

THE OPTIMAL ALLOCATION OF STATE AID IN NEW YORK

Introduction

In the previous chapter, a general model for the efficient allocation of State aid was presented as well as a model for predicting library use for multi-county areas. In this chapter we will frame the general allocation model in the context of one illustrative State, New York. The amount of State aid allocated to each of New York's library systems will be the critical decision variable in our model. The constraints placed upon our model will include measures of ability to pay, equity considerations, and a budget constraint. The parameters for each of these constraints will be estimated, and a solution to the mathematical model will be derived. A sensitivity analysis will be performed to indicate the degree to which the results depend on the accuracy of our estimates. Finally, the optimal values of State aid will be used as inputs to the prediction equation developed previously in order to judge the over-all "goodness" of our allocation precedures.

Mathematical Programming Formulation

The allocation model constructed in Chapter 3 can be solved as a linear programming problem if the following assumptions are satisfied:

(1) Proportionality. The objective function and

every constraint in a linear programming problem must be linear. Proportionality requires that the measure of effectiveness and resource usage must be proportional to the level of each activity conducted individually.

(2) Additivity. This requirement states that there are no interactions between the variables or activities with respect to either the measure of effectiveness or each resource usage. The activities are "additive," and a change in one activity level will have no effect on any other activity level.

(3) Divisibility. This requirement implies that fractional levels of the activity levels are permissible.

These assumptions imply in economic terms that the model exhibits constant returns to scale: doubling all inputs will yield a level of output which is exactly twice the previous level.

A strict interpretation of these assumptions would greatly limit the applicability of linear programming techniques. However, modest departures from these assumptions in the past have been found to be acceptable, and for purposes of this research these basic assumptions will be accepted as legitimate.

The Objective Function

The objective of the linear programming model is to maximize the value of $\sum a_i S_i$, where S_i is the amount of State aid allocated to library system i and a_i can be interpreted as the relative effectiveness of State aid in determining library use. In analogous economic terms, the coefficient a_i would be interpreted as the rate of return in library system i of one dollar of State aid. In the model these coefficients are measured in units of exposure counts per dollar of State aid. If we assume that all systems are equally effective in their use of State aid, then we would set $a_i = 1.0$ for all library systems. If we are unwilling to make this assumption, another procedure would be to estimate a_i from time series data for each library system.

The Constraints

We have placed two major constraints on the allocation procedure. The first constraint incorporates a measure of ability to pay into a requirement that minimum per capita levels of service must be satisfied. The second constraint places a budgetary limitation on the total amount of State aid which can be allocated to all library systems.

Ability to Pay

We shall now develop estimates of ability to pay for library service for each of New York State's library systems, based upon methodology that was developed in

Chapter 3. For each system, we shall call these measures "estimated capacity for library support." Four alternative measures will be constructed and analyzed. Summarizing the methodology, we will:

(1) identify the principal classes of revenue and tax bases that are utilized at the local level,

(2) obtain average tax rates for each tax class and the average library expenditure rate,

(3) multiply these average tax rates by the tax bases for each library system, and aggregate over all tax bases to arrive at the total fiscal capacity for each library system area,

(4) multiply the total fiscal capacity for each library system area by the average library expenditure rate to arrive at the estimated capacity for library support.

Local governments, in New York State as elsewhere, have at their disposal several alternative and complementary methods of raising revenue to finance governmental operations. Fundamentally, this revenue can be classified as either tax or non-tax revenue. Tax revenue is collected via instruments such as the income tax, sales and usage taxes, and property taxes. Non-tax revenue is collected largely by means of licenses, fees, and surpluses from governmental operations (such as State lotteries).

The largest category of revenue for local governments

is the property tax. In 1967 local governments in New York collected $5.4 billion in revenue. Property tax revenue accounted for 62% of this total; other tax revenue, 20%; and non-tax revenue, 18%.[1]

The breakdown of total local government revenue into three categories follows the pattern of the Census of Governments and is sufficient for our illustrative purposes. For other purposes a finer breakdown may be desirable. For example, property tax revenue is composed of three principal classes: non-farm residential property tax, business property tax, and farm property tax. Each of these classes of property is often taxed at different rates.

For purposes of this research, four major tax bases have been identified: taxable property valuation, personal income, retail trade sales, and population. The closest degree of association probably exists between property tax revenue and taxable property valuation, although, as mentioned earlier, property tax revenue is an aggregation of several different classes of property, each one of which may be taxed at a different rate. The other three tax bases are proxies which will be used to approximate the actual tax bases of non-property tax revenue (such as wage taxes, sales taxes, etc.) and non-tax revenue (charges and

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[1]See U.S. Bureau of the Census, 1967 Census of Governments: New York State (Volume 7, number 32).

miscellaneous revenue from fees, licenses, and surpluses of governmental operations). Correlation analysis will be used to detect any differences in the four alternative measures.

We shall now construct ability to pay measures for the twenty-two library systems in New York State. These library systems serve all of the State's sixty three counties. Each county is located in only one system except for Ulster County, part of which is served by the Mid-Hudson Library System and part by the Ramapo-Catskill Library System. The data base has been adjusted to reflect this fact.

The tax bases and revenues for all local governments in New York State are shown in Table 4.1.

The two most prevalent local taxes in New York State are the property tax and sales and use taxes. The tax bases most closely associated with these two taxes are total taxable property values and total retail trade sales. Non-tax revenue, which includes charges and miscellaneous revenue of local governments, is not closely linked with a tax base. A proxy tax base will be used, and for this purpose two alternative proxy tax bases will be tested: personal income and population.

The analysis will consider the following four measures:

Table 4.1

New York State Tax Bases and Revenues, 1967

Item	Amount (thousands of dollars)
Tax Bases	
Personal income	$68,915,000
Taxable property valuation	48,996,840
Retail trade sales	29,091,480
Revenues	
Property tax revenue	3,311,146
Non-property tax revenue	1,056,211
Non-tax revenue	991,803
Total local government revenue	5,359,159

Sources: U.S. Bureau of the Census, 1970 Census of Population; New York State Division of the Budget, New York State Statistical Yearbook - 1970; U.S. Bureau of the Census, 1967 Census of Governments: Taxable Property Values; U.S. Bureau of the Census, 1967 Census of Business, Retail Trade; U.S. Bureau of the Census, 1967 Census of Governments: New York State.

	Revenue Source	Revenue Base
Measure 1:	property tax	taxable property valuation
	non-property tax non-tax revenue	retail trade sales personal income
Measure 2:	property tax	taxable property valuation
	non-property tax non-tax revenue	retail trade sales population
Measure 3:	total local govt. revenue	personal income
Measure 4:	total local govt. revenue	taxable property valuation

The purpose of this analysis is to determine how sensitive the measures are to various proxy tax bases and to different levels of detail.

Table 4.2 presents for each library system in New York actual library support from local governmental units and four measures of estimated capacity for library support. The measures of capacity are calculated by applying statewide average tax and library expenditure rates to the local area's tax bases. Actual library support is defined as the actual expenditures by local governments to public library systems, including payments based on contractual arrangements. Table 4.3 presents the data in the form of measures of capacity as a percentage of actual library support. These tables indicate that for the majority of New York's library systems, the potential economic resources available to support library service are greater than actual library support. However, the measures indicate that five library

Table 4.2

Actual and Estimated Capacity for Library Support in New York, 1967

Library System	Actual Support[a]	Measure(1)	Measure(2)	Measure(3)	Measure(4)
Brooklyn	$ 7,273,548	10,461,630	10,312,110	9,348,656	12,226,950
Buffalo and Erie	4,684,554	2,697,024	2,791,956	3,142,125	2,299,206
Chatauqua-Cattaraugus	364,834	575,201	611,064	557,826	518,816
Chemung-Southern Tier	424,328	644,848	691,164	682,336	558,315
Clinton-Essex-Franklin	130,830	292,473	329,713	296,069	242,504
Finger Lakes	283,072	687,654	732,491	682,769	649,176
Four County	832,022	854,934	902,824	952,757	677,653
Mid-Hudson	547,582	1,046,684	1,117,621	1,216,767	880,719
Mid-York	740,336	918,376	978,900	996,947	791,483
Mohawk Valley	572,046	699,532	736,116	768,202	615,417
Nassau	8,919,890	4,620,528	4,346,493	6,152,029	3,683,838
New York	11,691,600	14,465,900	14,276,930	11,827,850	15,469,480
Nioga	780,832	866,901	903,030	893,751	820,104
North Country	444,642	686,547	759,075	675,058	605,669
Onondaga	1,007,862	1,108,752	1,147,359	1,341,364	874,333
Pioneer	2,575,575	2,478,849	2,509,079	2,924.134	2,049,553
Queens	7,377,949	8,031,820	7,918,197	7,092,743	9,276,517
Ramapo Catskill	1,569,379	1,250,796	1,338,400	1,235,136	1,142,850
Southern Adirondack	261,291	443,930	496,804	464,421	378,833
Suffolk	3,989,932	1,926,021	2,114,880	2,645,038	1,317,123
Upper Hudson	710,361	1,054,321	1,076,520	1,318,230	795,638
Westchester	4,529,517	3,899,274	3,621,362	4,427,776	3,837,937

[a] Actual support is defined as expenditures by local governmental units for libraries

Table 4.3

Estimated Capacity for Library Support in New York as a Percentage of Actual Library Support, 1967

Library System	Measure(1)	Measure(2)	Measure(3)	Measure(4)
Brooklyn	143.83	141.78	128.53	168.10
Buffalo and Erie	57.57	59.60	67.07	49.08
Chatauqua-Cattaraugus	157.66	167.49	152.90	142.21
Chemung-Southern Tier	151.97	162.88	160.80	131.58
Clinton-Essex-Franklin	223.55	252.02	226.30	185.36
Finger Lakes	242.93	258.77	241.20	229.33
Four County	102.75	108.51	114.51	81.45
Mid-Hudson	191.15	204.10	222.21	160.84
Mid-York	124.05	132.22	134.66	106.91
Mohawk Valley	122.29	128.68	134.29	107.58
Nassau	51.80	48.73	68.97	41.30
New York	123.73	122.11	101.17	132.31
Nioga	111.02	115.65	114.46	105.03
North Country	154.40	170.72	151.82	136.21
Onondaga	110.01	113.84	133.09	86.75
Pioneer	96.24	97.42	116.25	79.58
Queens	108.86	107.32	96.13	125.73
Ramapo Catskill	79.70	85.28	78.70	72.82
Southern Adirondack	169.90	190.13	177.74	144.98
Suffolk	48.27	53.01	66.29	33.01
Upper Hudson	148.42	151.54	185.57	112.00
Westchester	86.09	79.95	97.75	84.73

systems, Buffalo and Erie, Nassau, Ramapo-Catskill, Suffolk, and Westchester Library Systems, consistently support library service in excess of their potential support levels.

The four measures of estimated capacity for library support are all very highly correlated, as shown in Table 4.4. Some of these high correlation coefficients are not surprising. For example, the first two measures differ only in the choice of a proxy tax base for non-tax revenue, the least important source of local government revenue. The correlation coefficient 1.000 indicates perfect correlation. Of course there is some rounding error involved. The measures of ability to pay which are the least highly correlated are the last two measures (correlation coefficient equals .969). The high correlations indicate that for practical purposes there is very little difference between the four measures. The measures are consistent and, except perhaps for changes in scale, seem to be interchangeable.

The relationship between the measures of estimated capacity and actual library support is also very close, as shown in Table 4.5. However, the correlation coefficients relating actual support and the four measures of potential support are lower than any of the correlations among the four measures themselves. These correlation coefficients indicate a fairly strong relationship between actual

Table 4.4

Correlation Matrix for Measures of Estimated Capacity for Library Support

	Measure(1)	Measure(2)	Measure(3)	Measure(4)
Measure(1)	1.000	1.000	.985	.995
Measure(2)		1.000	.983	.995
Measure(3)			1.000	.969
Measure(4)				1.000

Table 4.5

Correlation Between Actual Library Support and Measures of Estimated Capacity for Library Support

	Measure(1)	Measure(2)	Measure(3)	Measure(4)
Actual Library Support	.915	.911	.961	.878

library support and the four measures of ability to pay. Local areas that contribute more heavily to the support of their public libraries also score highly on the measures of ability to pay, and vice versa.

In practice, the choice of one specific measure of ability to pay naturally will be subject to political influences and other factors not explicitly accounted for in this research. However, one result of this analysis has been to highlight the close interrelationships between several alternative measures. Some of these measures are composed of several parts and thus are more difficult to construct than others. Similarly, some of these measures, particularly those based on the property tax, are more influenced by institutional factors than others. For example, taxable property valuation is influenced by such institutional factors as the frequency of reassessment periods, equalization rates, and favorable assessment values given to some industrial firms to encourage expansion. Although the choice is somewhat arbitrary, for purposes of this research, measure (3) will be used as the estimated capacity for library support for each library system.

The tax base associated with this measure is personal income. States that have instituted an income tax would normally collect these data as part of their tax collection

function. An alternative source of data on personal income by county of residence is the Internal Revenue Service. Changes in income are reflected in the tax base at annual intervals; changes in other tax bases, like taxable property valuation, occur only at infrequent intervals such as reassessment periods. Thus, the measure of ability to pay for multi-county systems of public libraries is based upon a fairly reliable statistical source, personal income, and the data base usually is or can be revised annually.

The measure of ability to pay for library service for New York State's library system areas is shown in column (4) of Table 4.2. This measure will be incorporated as one part of the equity constraint of our mathematical programming model. Another part of the equity constraint is the total cost of providing a minimum acceptable level of library service for each multi-county library system area. In Chapter 3 we denoted this quantity mP_i, where m is the cost per capita of providing a minimum acceptable level of library service, and P_i is the population of library system i. For New York, the population of each library system area is shown in Table 2.4.

The value of the parameter m is subject to many influences. First, the definition of a minimum acceptable level of service will surely change with the passage of time. In the beginning, rising expectations of the ability of libra-

ries to respond to the public's traditional demands will necessitate raising the minimum acceptable levels of service. In the future, as libraries continue to develop and grow and as the functions of the public library evolve in new directions, new minimum levels of service will have to be defined. Second, the cost per capita of providing a particular level of service may vary in different parts of a State. For example, salary costs in urban areas would generally be higher than in rural or suburban areas. However, there may be offsetting economies of scale in an urban area which lower the cost per capita of providing the minimum level of service. One of the extensions of this research is a deeper analysis of these costs and how they can be estimated accurately.

One attempt to establish norms for the cost of public library service was prepared by a subcommittee of the Public Library Association.[2] Under the direction of Eleanor Plain, this group presented a report which estimated the expected annual operating cost for libraries seeking to meet the requirements of the ALA's Minimum Standards for Public Library Systems, 1966. Using the same unit costs for salaries and library materials, sample budgets were constructed for library systems serving a population of 200,000, a

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[2]Eleanor Plain, "Costs of Public Library Service 1968," ALA Public Library Association, Vol. 7, No. 3, October, 1968

population of 600,000, and a population of 1,000,000. The annual cost per capita of meeting the ALA's minimum standards was estimated to be:

(1) $5.80 for a library system serving 200,000 people,

(2) $5.36 for a system serving 600,000 people, and

(3) $5.31 for a system serving 1,000,000 people.

The average cost per capita for all three illustrative library systems was approximately $5.50 per capita, based on 1968 prices and wage rates.

Clearly these standard budgets become outdated very quickly. An earlier estimate based on 1962 prices and wage rates calculated the average cost of meeting the minimum standards as $3.82 per capita.[3] Compare these figures with a more recent analysis which revises the unit cost figures to a range of $7.66 to $8.23 per capita for the year 1971.[4]

Estimates of the average cost of providing adequate library services are subject to serious questions. For instance, the criticism that the definition of "minimum levels of services" promulgated by the ALA is unrealistically high cannot be rebutted by the evidence. In 1965, "ninety-seven percent of the public libraries failed to meet the standard for general operating expenditures.... In 25 out of 50

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[3] Henry T. Drennan, "The Public Library Service Gap," in National Inventory of Library Needs, Chicago, American Library Association, 1965, p. 40.

[4] Lowell Martin, "Standards for Public Libraries," Library Trends, 21:164-77 (October, 1972), p. 167.

states no public library met this standard."[5]

In lieu of an authoritative statement concerning the appropriate value for m, we will consider m to be a policy variable subject to control by State library administrators. Library administrators can determine the effect of any particular value of m on the allocation of State aid by using the mathematical programming model. A later section in the chapter examines the results of the allocation procedure for different values of m.

Budget Constraint

The second type of constraint in the allocation model is the budget. This constraint places a ceiling on the total amount of funds which are available for State aid. The value for this variable in the model is the actual expenditure level for New York State, $14,456,290. Other budgetary ceilings could be substituted for this value and the results of the model with different sizes of budgets could be compared. In a later section of this chapter, sensitivity analysis is performed on the budget constraint to determine what changes will occur when changes are made in this constraint.

The models described in Chapter 3 have now been presented with respect to New York State. The remaining parts of this chapter deal with the results and validation of the

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[5] Drennan, "The Public Library Service Gap," p.41.

mathematical programming model. In particular, extensive work will be reported on the sensitivity of the model to various changes in the model's parameters.

The basic linear programming model which will be studied in this section is the following:

$$\text{Maximize: } Z = \sum_{i=1}^{22} a_i S_i$$

$$\text{subject to: } S_i \geq mP_i - Y_i \quad \text{for } i = 1,2,\ldots,22$$

$$\sum_{i=1}^{22} S_i \leq T$$

$$S_i \geq 0 \quad \text{for } i = 1,2,\ldots,22$$

The notation and definition of terms is the same as Chapter 3 and is summarized in the glossary.

Initial Solution of the Model

We have described earlier the estimation or choice of the values of the parameters a_i, Y_i, m, P_i, and T. The initial solution represents the optimal allocation of funds using the current value for T and an approximation to the current value for m. In 1967, total operating expenditures for libraries excluding State aid amounted to $64,177,438, and the State's total population according to the 1970 Census of Population was 18,190,740. An approximation for m, the cost of meeting the minimum acceptable levels of service, is the ratio, $m = \frac{\$64,177,438}{18,190,740} = \$3.53$. If we let

$m = \$3.50$

$T = \$14,456,290$

$$a_i = 1.0 \quad \text{for } i = 1,2,\ldots,22$$

and let Y_i and P_i be the values shown in column (4) of Table 4.2 and Table 2.4, respectively, the solution to this linear programming problem is shown in Table 4.7.* For this and subsequent tables, section 1 of the table refers to the row constraints. Row numbers 2 through 23 are associated with the equity constraint for each library system. The system abbreviations are identified in Table 4.6. The abbreviation "TOT" refers to the budget constraint. Section 2 of the table refers to the variables in the problem. The variables are the amount of State aid received by each system, denoted by the symbols S1 to S22. The translation from symbol to system is given in Table 4.6.

Assuming that all systems are equally effective in providing library service per dollar of State aid, and assuming that $3.50 per capita is a reasonable approximation for the cost of providing a minimum level of library service, then the optimal allocation of the State aid budget can include as few as seventeen of the State's library systems. The five systems whose estimated ability to support library service exceeds their needs are: Brooklyn, Nassau, New York, Queens, and Westchester County Library Systems.

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*The values shown in this and subsequent tables were computed with the use of IBM's Mathematical Programming System/360.

Table 4.6

New York Library System Abbreviations and Sysbols for State Aid

Library System	Abbreviation	Symbol for State Aid
Brooklyn	BN	S1
Buffalo-Erie	BE	S2
Chatauqua-Cattaraugus	CC	S3
Chemung-Southern Tier	CST	S4
Clinton-Essex-Franklin	CEF	S5
Finger Lakes	FL	S6
Four County	FC	S7
Mid-Hudson	MH	S8
Mid-York	MY	S9
Mohawk Valley	MV	S10
Nassau	NAS	S11
New York	NY	S12
Nioga	NIO	S13
North Country	NC	S14
Onondaga	O	S15
Pioneer	P	S16
Queens	Q	S17
Ramapo Catskill	RAM	S18
Southern Adirondack	SA	S19
Suffolk	SU	S20
Upper Hudson	UH	S21
Westchester	W	S22

Table 4.7

Solution to Allocation Model Using m= $3.50, a = 1.0

Section 1 - Rows

	Number	Row	At	Activity	Slack Activity	Lower Limit	Upper Limit	Dual Activity
	1	VALUE	BS	14456.290	14456.290-	NONE	NONE	1.000
	2	BN	BS	.	.	.	NONE	.
	3	BE	BS	8043.979	7288.885	755.094	NONE	.
A	4	CC	LL	243.573	.	243.573	NONE	.
A	5	CST	LL	312.046	.	312.046	NONE	.
A	6	CEF	LL	234.167	.	234.167	NONE	.
A	7	FL	LL	303.559	.	303.559	NONE	.
A	8	FC	LL	339.030	.	339.030	NONE	.
A	9	MH	LL	489.301	.	489.301	NONE	.
A	10	MY	LL	414.747	.	414.747	NONE	.
A	11	MV	LL	261.670	.	261.670	NONE	.
	12	NAS	BS	.	.	.	NONE	.
	13	NY	BS	.	.	.	NONE	.
A	14	NIO	LL	267.364	.	267.364	NONE	.
A	15	NC	LL	462.582	.	462.582	NONE	.
A	16	O	LL	311.284	.	311.284	NONE	.
A	17	P	LL	372.513	.	372.513	NONE	.
	18	Q	BS	.	.	.	NONE	.
A	19	RAM	LL	586.544	.	586.544	NONE	.
A	20	SA	LL	335.399	.	335.399	NONE	.
A	21	SU	LL	1263.314	.	1263.314	NONE	.
A	22	UH	LL	215.218	.	215.218	NONE	.
	23	W	BS	.	.	.	NONE	.
	24	TOT	UL	14456.290	.	NONE	14456.290	1.000-

Table 4.7 (continued)

Section 2 - Columns

	Number	Column	At	Activity	Input Cost	Lower Limit	Upper Limit	Reduced Cost
A	25	S1	LL	.	1.000	.	NONE	.
	26	S2	BS	8043.979	1.000	.	NONE	.
	27	S3	BS	243.573	1.000	.	NONE	.
	28	S4	BS	312.046	1.000	.	NONE	.
	29	S5	BS	234.167	1.000	.	NONE	.
	30	S6	BS	303.559	1.000	.	NONE	.
	31	S7	BS	339.030	1.000	.	NONE	.
	32	S8	BS	489.301	1.000	.	NONE	.
	33	S9	BS	414.747	1.000	.	NONE	.
	34	S10	BS	261.670	1.000	.	NONE	.
A	35	S11	LL	.	1.000	.	NONE	.
A	36	S12	LL	.	1.000	.	NONE	.
	37	S13	BS	267.364	1.000	.	NONE	.
	38	S14	BS	462.582	1.000	.	NONE	.
	39	S15	BS	311.284	1.000	.	NONE	.
	40	S16	BS	372.513	1.000	.	NONE	.
A	41	S17	LL	.	1.000	.	NONE	.
	42	S18	BS	586.544	1.000	.	NONE	.
	43	S19	BS	335.399	1.000	.	NONE	.
	44	S20	BS	1263.314	1.000	.	NONE	.
	45	S21	BS	215.218	1.000	.	NONE	.
A	46	S22	LL	.	1.000	.	NONE	.

The entire budget can be allocated to the seventeen systems to meet their minimum levels of service and a surplus of $7,288,885 will remain.

Let us explain in more detail how we interpret these results. Section 2 of Table 4.7 identifies the five systems that receive no State aid: S1, S11, S12, S17, and S22. These refer to Brooklyn, Nassau, New York, Queens, and Westchester Library Systems. Section 1 of Table 4.7 shows the activity of each system and the budget constraint, as well as other minor bits of information. The budget constraint (symbolized by "TOT") is fully satisfied, and each library system at least meets its minimum requirements for State aid. Buffalo-Erie Library System receives $7,288,885 more than its minimum requirement of $755,094. In this research we define surplus as the difference between the total State aid budget and the sum of the minimum amounts of State aid necessary to provide adequate levels of service for each library system. Thus the value of the surplus in this model is $7,288,885. The computer algorithm has arbitrarily assigned this surplus to the first row that has a non-zero activity level, Buffalo-Erie.

A consequence of the assumption that all systems are equally effective in providing library service per dollar of State aid is that the solution will have alternate optima. These are indicated in Table 4.7 when the letter

"A" appears to the left of a variable. In this case each of the five library systems not included in the final solution is an alternate optimum. Thus, each of these systems individually, or in combination, can be included in the basis without changing the value of the objective function. The letter "A" to the left of a row in Section 1 of Table 4.7 indicates which variable can be replaced in the basis by one of the alternates. In this case every row, with the exception of Buffalo-Erie, fits into this category. Therefore, a very large number of combinations of variables could comprise an optimal solution to this model. This situation can be exploited in a useful and important way. If there were political pressures to grant some State aid to every library system, then we can distribute the surplus $7,288,885 among the five remaining library systems. One possible split is to grant equal shares to all five systems, or $1,457,777 to each. Let us denote this policy P(1). Another possibility is to grant a larger proportion of the surplus to the three systems which comprise New York City - Brooklyn, New York, and Queens Library Systems. Suppose that 25 per cent of the surplus is allocated to each of these three systems, and the remaining 25 per cent is evenly distributed to Nassau and Westchester County. Then the amounts received by these systems would be:

Brooklyn	$1,822,221
New York	1,822,221
Queens	1,822,221
Nassau	911,111
Westchester	911,111
	$7,288,885

Let us denote this latter policy for State-aid distribution as Policy P(2).

A comparison of the results of the allocation model assuming m = $3.50 with the actual distribution of State aid reveals many interesting comparisons, as shown in Table 4.8. Disregarding the many possible ways to distribute surplus State aid, twelve library systems would receive less State aid using the linear programming model than they actually received. Naturally ten systems would receive more State aid. If we calculate the percentage of estimated State aid to actual aid, thirteen library systems receive between 80 to 120 percent of their actual aid, a fact which seems particularly surprising since about half of the total State aid budget in the model is undistributed surplus. As Table 4.8 shows, if we distribute this surplus evenly among the library systems which receive no State aid, then nine systems would receive less State aid using the allocation model than they actually received, and thirteen systems would receive more State aid. Further-

Table 4.8

Comparison of Actual State Aid with Results of Allocation Model Using m = $3.50

System	Actual State Aid (thousand dollars)	Minimum State Aid, m = $3.50	Percentage, Min. to Actual State Aid	Percentage, if Surplus Distributed Using Policy P(1)	Percentage, if Surplus Distributed Using Policy P(2)
BN	1605.08	0	0	90.82	113.53
BE	865.55	755.09	87.24	87.24	87.24
CC	243.43	243.57	100.06	100.06	100.06
CST	280.98	312.05	111.06	111.06	111.06
CEF	209.23	234.17	111.92	111.92	111.92
FL	316.79	303.56	95.82	95.82	95.82
FC	413.28	339.03	82.03	82.03	82.03
MH	356.49	489.30	137.26	137.26	137.26
MY	346.46	414.75	119.71	119.71	119.71
MV	266.23	261.67	98.29	98.29	98.29
NAS	1036.99	0	0	140.58	113.82
NY	2888.90	0	0	50.46	63.07
NIO	333.32	267.36	80.21	80.21	80.21
NC	422.60	462.58	109.46	109.46	109.46
O	294.01	311.28	105.88	105.88	105.88
P	795.94	372.51	46.80	46.80	46.80
Q	1373.09	0	0	106.17	132.71
RAM	460.66	586.54	127.33	127.33	127.33
SA	292.12	335.40	114.82	114.82	114.82
SU	782.61	1263.31	161.42	161.42	161.42
UH	266.26	215.22	80.83	80.83	80.83
W	607.18	0	0	240.09	150.06

more, fifteen library systems would receive between 80 and 120 percent of their actual aid. Two library systems most drastically affected by this particular allocation scheme are Westchester and New York. According to the model, neither of these systems needs to receive any State aid. However, if the surplus is evenly distributed according to policy P(1), Westchester would receive 240 percent more aid than it actually received, and New York would receive only 50 percent of its actual aid. Considering only the systems that receive State aid, the one that receives the least State aid using the allocation model relative to its current level is Pioneer Library System. The allocation model would reduce this system's aid by more than half from $795,940 to $372,510.

If the surplus were distributed according to policy P(2) (which allocates more State aid to the three highly-populated metropolitan New York library systems), then the variation between estimated and actual State aid is diminished. Under policy P(2) the percentage for all systems ranges from the lowest 46.80 percent for Pioneer to the highest 150.06 percent for Westchester Library System. Fifteen library systems would receive between 80 and 120 percent of their actual State aid.

Parametric Study of m

In this section we will examine the behavior of the

allocation model when different values for m are chosen. Recall that in the initial formulation of the model, the policy variable m was set equal to $3.50, a value that closely approximated the actual average level of library expenditures per capita in New York State.

If this level of support were considered to be inadequate to meet the minimum standards of library service in New York, then we can change the value of m and examine how the new value affects the amount of State aid distributed to each library system. Suppose that m were increased to $4.00 per capita, an increase of almost 15 per cent, and suppose that there are no other changes in the model's parameters. The consequences of this change on the optimal solution are displayed in Table 4.9.

As the table indicates, the State aid budget of $14,456,290 can be allocated in an optimal manner by distributing the entire amount among twenty of the twenty-two library systems. After meeting the minimum requirements of aid in each system, there will be a surplus of $92,765. The two library systems whose local resources exceed the cost of providing minimum levels of service are Nassau and Westchester County Library Systems. However, as Section 2 of Table 4.9 indicates, both of these library systems are alternate optima and could be included in the basis without changing the value of the objective function.

Table 4.9

Solution to Allocation Model, m= $4.00, a = 1.0

Section 1 - Rows

	Number	Row	At	Activity	Slack Activity	Lower Limit	Upper Limit	Dual Activity
	1	VALUE	BS	14456.290	14456.290-	NONE	NONE	1.000
	2	FN	BS	1141.517	92.765-	1048.752	NONE	.
A	3	BE	LL	1311.839	.	1311.839	NONE	.
A	4	CC	LL	358.058	.	358.058	NONE	.
A	5	CST	LL	454.100	.	454.100	NONE	.
A	6	CEF	LL	309.915	.	309.915	NONE	.
A	7	FL	LL	444.463	.	444.463	NONE	.
A	8	FC	LL	523.571	.	523.571	NONE	.
A	9	MH	LL	733.025	.	733.025	NONE	.
A	10	MX	LL	616.417	.	616.417	NONE	.
A	11	MV	LL	408.794	.	408.794	NONE	.
	12	NAS	BS	.	.	.	NONE	.
A	13	NY	LL	1340.950	.	1340.950	NONE	.
A	14	NIO	LL	433.237	.	433.237	NONE	.
A	15	NC	LL	625.102	.	625.102	NONE	.
A	16	O	LL	547.378	.	547.378	NONE	.
A	17	P	LL	853.462	.	853-462	NONE	.
A	18	Q	LL	802.089	.	802.089	NONE	.
A	19	RAM	LL	846.784	.	846.784	NONE	.
A	20	SA	LL	449.659	.	449.659	NONE	.
A	21	SU	LL	1821.650	.	1821.650	NONE	.
A	22	UH	LL	434.282	.	434.282	NONE	.
	23	W	BS	.	.	.	NONE	.
	24	TOT	UL	14456.290	.	NONE	14456.290	1.000-

Table 4.9 (continued)

Section 2 - Columns

	Number	Column	At	Activity	Input Cost	Lower Limit	Upper Limit	Reduced Cost
	25	S1	BS	1141.517	1.000	.	NONE	.
	26	S2	BS	1311.839	1.000	.	NONE	.
	27	S3	BS	358.058	1.000	.	NONE	.
	28	S4	BS	454.100	1.000	.	NONE	.
	29	S5	BS	309.915	1.000	.	NONE	.
	30	S6	BS	444.463	1.000	.	NONE	.
	31	S7	BS	523.571	1.000	.	NONE	.
	32	S8	BS	733.025	1.000	.	NONE	.
	33	S9	BS	616.417	1.000	.	NONE	.
	34	S10	BS	408.794	1.000	.	NONE	.
A	35	S11	LL	.	1.000	.	NONE	.
	36	S12	BS	1340.950	1.000	.	NONE	.
	37	S13	BS	433.237	1.000	.	NONE	.
	38	S14	BS	625.102	1.000	.	NONE	.
	39	S15	BS	547.376	1.000	.	NONE	.
	40	S16	BS	853.462	1.000	.	NONE	.
	41	S17	BS	802.089	1.000	.	NONE	.
	42	S18	BS	846.784	1.000	.	NONE	.
	43	S19	BS	449.659	1.000	.	NONE	.
	44	S20	BS	1821.650	1.000	.	NONE	.
	45	S21	BS	434.282	1.000	.	NONE	.
A	46	S22	LL	.	1.000	.	NONE	.

A value of m = $4.00, therefore, is sufficient to reduce the size of the surplus from $7,288,885 to $92,765, an amount that represents less than 1 per cent of the total State-aid budget. In the previous model the average percentage of minimum State aid to actual State aid equaled 104.13 per cent. The new model produces higher levels of minimum State aid, as shown in Tables 4.9 and 4.10. A comparison of these levels with actual State aid reveals that of the library systems receiving aid, only three would receive less than their actual amount of State aid: Brooklyn, New York, and Queens, the New York City systems. The average percentage of minimum to actual State aid for the systems receiving aid is 144.37. As shown in Table 4.10, this percentage ranges from a low of 46.42 per cent for New York to a high of 232.77 per cent for Suffolk Library System.

Unlike the initial model, a policy to distribute the surplus among the systems that do not receive any aid does not substantially affect the results. If the surplus were allocated equally to Nassau and Westchester County Library Systems, it would represent only 4.47 and 7.64 per cent, respectively, of their actual State aid.

Suppose that policy makers at the State level have determined that the value of m should be increased to $4.50 per capita. Assuming no increase in the total budget and

Table 4.10

Comparison of Actual State Aid with the Results of the Model
Using m = $3.50 and m = $4.00

System	Actual State Aid (thousand dollars)	Minimum State Aid m = $3.50	Minimum State Aid m = $4.00	Percentage, Minimum to Actual State Aid (m = $4.00)
BN	1605.08	0	1048.75	65.34
BE	865.55	755.09	1311.84	151.56
CC	243.43	243.57	358.06	147.09
CST	280.98	312.05	454.10	161.61
CEF	209.23	234.17	309.92	148.12
FL	316.79	303.56	444.46	140.30
FC	413.28	339.03	523.57	126.69
MH	356.49	489.30	733.03	205.62
MY	346.46	414.75	616.42	177.92
MV	266.23	261.67	408.79	153.55
NAS	1036.99	0	0	0
NY	2898.90	0	1340.95	46.42
NIO	333.32	267.36	433.24	129.98
NC	422.60	462.58	625.10	147.92
O	294.01	311.28	547.38	186.18
P	795.94	372.51	853.46	107.23
Q	1373.09	0	802.09	58.41
RAM	460.66	586.54	846.78	183.82
SA	292.12	335.40	449.66	153.93
SU	782.61	1263.31	1821.65	232.77
UH	266.26	215.22	434.28	163.10
W	607.18	0	0	0

assuming that all systems are equally effective in providing library service per dollar of State aid, then the only changes in the allocation model are in the values of the right-hand side coefficients for each library system. The consequence of this change in the policy variable m is that there is no feasible allocation of State aid which will satisfy all the constraints. Given a limited budget, the State cannot meet the expanded cost implied by the higher levels of service. Either the value of m will have to be scaled downward, or the budget will have to be increased.

Parametric Study of Budget Constraint

In the initial allocation model, it was assumed that:

(1) m = $3.50 per capita represented a reasonable approximation to the cost of providing a minimum level of library service;

(2) all library systems were equally effective in providing library service per dollar of State aid; and

(3) the size of the State-aid budget T = $14,456,290.

The optimum solution showed that the size of the budget was not a limiting factor in the allocation process. In fact, more than 50 per cent of the budget can be considered unallocated or undivided surplus. Given the large number of alternate optima, the surplus could be divided in an infinite number of ways. Examples of two possible policies for distributing the surplus were given earlier. An increase of one dollar in the size of the budget in this

model will have the effect of increasing the size of the surplus by one dollar.

If the value of m were set equal to $4.00, the result would be similar because the budget constraint in that model was shown not to be the limiting factor in the allocation process. Although the amount of the undivided surplus was small (less than 1 per cent of the total budget), a one dollar increase in the size of the budget will increase the size of the surplus by one dollar.

The budget constraint does limit the distribution of State aid when the value of m is increased to $4.50. As described previously, this value for m resulted in an infeasible solution of the allocation model. The amount of State-aid funds available was not sufficient to meet the higher minimum levels of service represented by this value of m. Table 4.11 presents the amount of State aid necessary to meet these levels of service for each library system. The budget constraint would have to be increased from the present level of $14,456,290 to $22,563,510 in order to satisfy all the constraints. Thus an increase of only $0.50 per capita in the value of m necessitates an increase in the size of the budget by more than 56 per cent.

The conclusions that can be drawn from the allocation model at this time are:

(1) assuming that $a_i = 1.0$ for all systems, then

Table 4.11

Minimum Levels of State Aid when m = $4.50

Library System	Minimum State Aid, m = $4.50 (Thousand dollars)
Brooklyn	$2,359.69
Buffalo-Erie	1,868.59
Chatauqua-Cattaraugus	472.54
Chemung-Southern Tier	596.16
Clinton-Essex-Frnaklin	385.66
Finger Lakes	585.37
Four County	708.11
Mid-Hudson	976.75
Mid-York	818.81
Mohawk Valley	555.92
Nassau	251.04
New York	2,987.05
Nioga	599.11
North Country	787.62
Onondaga	783.47
Pioneer	1,334.41
Queens	1,788.94
Ramapo Catskill	1,107.02
Southern Adirondack	563.92
Suffolk	2,379.99
Upper Hudson	653.35
Westchester	0
Total	$ 22,563.51

an increase in the size of the budget will lead to an increase in the amount of undistributed surplus, provided that the initial solution is feasible.

(2) small changes in the policy variable m have relatively great impact on the feasibility of the solution to the allocation model; changes in m directly affect each right-hand side coefficient, and the cumulative effect on the size of the budget T can be considerable.

Parametric Study of Objective Function

In the initial formulation of the allocation model, it was assumed that all coefficients in the objective function were equal to 1.0. This assumption symbolizes the view that all library systems are equally effective in utilizing State aid to produce document exposure. These coefficients are measured in units of exposure counts per dollar of State aid. In this section we will discuss two basic questions:

(1) What is the effect of slight variations in the values of a_i ?

(2) What is the effect of alternative formulations of the objective function?

It was observed that choosing all the objective function coefficients to be equal to 1.0 led to a large number of alternate optima. In the case of m = $3.50, five

variables could be included in the basis without changing the value of the objective function, while in the case of m = $4.00 two variables fit into this category. The existence of alternate optima can be quite useful for policy purposes. As shown earlier, various policies can be developed when alternate optima exist that greatly increase the range of alternative courses of action available to State-level library administrators and budget officials. The two policies that were illustrated were: (a) division of the "surplus" in equal shares to systems receiving no State aid; and (b) division of the "surplus" in a way that provided more State aid to the three systems comprising New York City.

Slight variations in the values of a_i will result in important changes in the solution of the allocation model. For example, if the coefficient for Nassau County were increased from 1.0 to 1.1 and there were no changes made in the values of the other coefficients, then Nassau would receive the entire "surplus." A similar statement could be made for every other library system. Thus, the objective function coefficients need to be estimated very precisely. Unless further constraints are placed on the allocation procedure, the library system which is relatively most efficient in providing library service per dollar of State aid will receive all the budgetary "surplus."

Let us consider an alternative formulation of the objective function. Suppose that instead of the assumption that all library systems are equally effective in providing service per dollar of State aid, we assume that there are distinct differences among library systems. Furthermore, let us assume that the measure of effectiveness for each library system is equal to the ratio of annual number of exposure counts per dollar of State aid. For 1968 this ratio ranges from a low of 2.765 to a high of 10.744 exposures per dollar of State aid, as shown in Table 4.12.

This formulation of the values of the coefficients in the objective function results in an allocation of State aid illustrated by Table 4.13. Using a value for m equal to $4.00 per capita, we find that the entire "surplus" is allocated to Westchester Library System. Furthermore, in this formulation there are no alternate optima (Nassau Library System receives no State aid). Similarly, had we chosen m to be $3.50 per capita, the budget "surplus" still would be allocated to Westchester Library System; neither Brooklyn, Nassau, New York, nor Queens Library System would receive any State aid, and there would be no alternate optima.

If it is felt that the ratio of number of exposure counts to State aid does not represent a true measure of a library system's effectiveness, then another method of

Table 4.12

Number of Exposures Counts and State Aid for New York State's Library Systems, 1968

Library System	Number of Exposure Counts, C	State Aid, S	Ratio, C/S
Brooklyn	8,211,127	$1,605,082	5.116
Buffalo and Erie	6,478,226	865,547	7.485
Chatauqua-Cattaraugus	1,319,094	243,434	5.419
Chemung-Southern Tier	1,286,697	280,983	4.579
Clinton-Essex-Franklin	646,878	209,225	3.092
Finger Lakes	1,101,536	316,787	3.477
Four County	1,521,523	413,281	3.682
Mid-Hudson	1,375,249	356,492	3.858
Mid-York	1,515,391	346,459	4.374
Mohawk Valley	1,339,735	266,230	5.032
Nassau	9,949,709	1,036,991	9.595
New York	12,195,149	2,888,898	4.221
Nioga	1,385,004	333,322	4.155
North Country	1,168,701	422,595	2.765
Onondaga	2,360,171	294,005	8.028
Pioneer	4,899,866	795,944	6.156
Queens	8,115,108	1,373,088	5.910
Ramapo Catskill	1,971,505	460,664	4.280
Southern Adirondack	872,307	292,116	2.986
Suffolk	4,907,337	782,607	6.270
Upper Hudson	1,382,321	266,257	5.192
Westchester	6,521,432	607,184	10.744

Table 4.13

Solution to Allocation Model, m= \$4.00, $a_i = C_i / S_i$

Section 1 - Rows

Number	Row	At	Activity	Slack Activity	Lower Limit	Upper Limit	Dual Activity
1	VALUE	BS	14456.290	14456.290-	NONE	NONE	1.000
2	BN	LL	1048.752	.	1048.752	NONE	5.628
3	BE	LL	1311.839	.	1311.839	NONE	3.259
4	CC	LL	358.058	.	358.058	NONE	5.325
5	CST	LL	454.100	.	454.100	NONE	6.165
6	CEF	LL	309.915	.	309.915	NONE	7.652
7	FL	LL	444.463	.	444.463	NONE	7.267
8	FC	LL	523.571	.	523.571	NONE	7.062
9	MH	LL	733.025	.	733.025	NONE	6.886
10	MY	LL	616.417	.	616.417	NONE	6.370
11	MV	LL	408.794	.	408.794	NONE	5.712
12	NAS	BS	.	.	.	NONE	.
13	NY	LL	1340.950	.	1340.950	NONE	6.523
14	NIO	LL	433.237	.	433.237	NONE	6.589
15	NC	LL	625.102	.	625.102	NONE	7.979
16	O	LL	547.376	.	547.376	NONE	2.716
17	P	LL	853.462	.	853.462	NONE	4.588
18	Q	LL	802.089	.	802.089	NONE	4.834
19	RAM	LL	846.784	.	846.784	NONE	6.464
20	SA	LL	449.659	.	449.659	NONE	7.758
21	SU	LL	1821.650	.	1821.650	NONE	4.474
22	UH	LL	434.282	.	434.282	NONE	5.552
23	W	BS	92.765	92.765-	.	NONE	.
24	TOT	UL	14456.290	.	NONE	14456.290	10.744

Table 4.13 (continued)

Section 2 - Columns

Number	Column	At	Activity	Input Cost	Lower Limit	Upper Limit	Reduced Cost
25	S1	BS	1048.752	5.116	.	NONE	.
26	S2	BS	1311.839	7.485	.	NONE	.
27	S3	BS	358.058	5.419	.	NONE	.
28	S4	BS	454.100	4.579	.	NONE	.
29	S5	BS	309.915	3.092	.	NONE	.
30	S6	BS	444.463	3.477	.	NONE	.
31	S7	BS	523.571	3.682	.	NONE	.
32	S8	BS	733.025	3.858	.	NONE	.
33	S9	BS	616.417	4.374	.	NONE	.
34	S10	BS	408.794	5.032	.	NONE	.
35	S11	LL	.	9.595	.	NONE	1.149-
36	S12	BS	1340.950	4.221	.	NONE	.
37	S13	BS	433.237	4.155	.	NONE	.
38	S14	BS	625.102	2.765	.	NONE	.
39	S15	BS	547.376	8.026	.	NONE	.
40	S16	BS	853.462	6.156	.	NONE	.
41	S17	BS	802.089	5.910	.	NONE	.
42	S18	BS	846.784	4.280	.	NONE	.
43	S19	BS	449.659	2.986	.	NONE	.
44	S20	BS	1821.650	6.270	.	NONE	.
45	S21	BS	434.282	5.192	.	NONE	.
46	S22	BS	92.765	10.744	.	NONE	.

estimating the coefficients a_i must be found. One suggested procedure is to estimate these coefficients for each library system from appropriate time series data. Unfortunately the statistical series for New York State's library systems is not sufficiently lengthy to warrant that kind of analysis.

Combination of the Allocation Model with Prediction Equation

The results of the allocation model provide the State level administrator with a set of optimal values for State aid, S_i^*. These values can be linked with the prediction model developed in Chapter 3 in order to translate any value of S_i^*, representing the optimal amount of State aid received by library system i, into a predicted level of library use. Mathematically, we found in Chapter 3 that a strong linear relationship exists between the number of exposure counts C and

(1) State aid, S

(2) annual change in local government expenditures for libraries, ΔG, and

(3) annual change in number of books added by local purchase and gifts, ΔK.

The multiple regression equation was estimated as:

$$C = \underset{(2.265)}{-3.475} + \underset{(0.582)}{4.463}\,\Delta G - \underset{(0.062)}{.309}\,\Delta K + \underset{(0.025)}{.495}\,S$$

The variable ΔG is determined exogenously by various local

governments while ΔK is determined by the actions of the specific library system. Given values for ΔG and ΔK for library system i, then the predicted number of exposure counts when State aid is S_i^* is denoted $\hat{C}_i$.

Suppose that State administrators are willing to make the following assumptions:

(1) $m = \$3.50$ per capita,

(2) $a_i = 1.0$ for all library systems, and

(3) the budget "surplus" is equally distributed among the systems receiving no State aid (denoted policy P(1)).

The question arises, "What are the predicted levels of library use if State aid were allocated on the basis of these assumptions?"

The answer to this question is displayed in Table 4.14 under policy P(1). The predicted levels of library use range from 871,000 exposure counts for Clinton-Essex-Franklin Library System to 12,117,000 exposure counts for Nassau Library System. A comparison between the predicted and actual number of exposure counts indicates that the largest prediction errors occur in the library systems that comprise New York City and surrounding areas. The differences between the predicted and actual values for these systems are:

Table 4.14

Comparison of Predicted Number of Exposures Using Policies P(1) and P(2) with Actual Number of Exposures

	Actual Number of Exposures (1968) (thousands)	Predicted Number of Exposures Policy P(1) (thousands)	Predicted Number of Exposures Policy P(2) (thousands)
Brooklyn	8,211	7,418	9,221
Buffalo and Erie	6,478	5,268	5,268
Chatauqua-Cattaraugus	1,319	1,195	1,195
Chemung-Southern Tier	1,287	1,400	1,400
Clinton-Essex-Franklin	647	871	871
Finger Lakes	1,102	1,306	1,306
Four County	1,522	1,400	1,400
Mid-Hudson	1,375	1,965	1,965
Mid-York	2,515	1,843	1,843
Mohawk Valley	1,340	894	894
Nassau	9,950	12,117	9,412
New York	12,195	5,603	7,406
Nioga	1,385	2,064	2,064
North Country	1,169	2,233	2,233
Onondaga	2,360	1,466	1,466
Pioneer	4,900	3,005	3,005
Queens	8,115	7,220	9,023
Ramapo Catskill	1,972	2,371	2,371
Southern Adirondack	872	1,498	1,498
Suffolk	4,907	8,509	8,509
Upper Hudson	1,382	1,111	1,111
Westchester	6,521	9,761	7,056

	Prediction error (thousands)
Brooklyn	- 793
Nassau	2,167
New York	-6,592
Queens	- 895
Suffolk	3,602
Westchester	3,240

Policy P(2) provides an alternative method of distributing the "surplus" State aid in the allocation model. This policy deals with all library systems listed above except for Suffolk, and under this policy more State aid is provided to urban systems as opposed to the suburban systems. The predicted number of exposures using policy P(2) is displayed in the last column of Table 4.14. The prediction errors for the five relevant library systems are:

	Prediction error (thousands)
Brooklyn	1,010
Nassau	- 538
New York	-4,789
Queens	908
Westchester	535

Policy P(2) is somewhat more representative of current State policy than P(1), since the prediction errors generally are smaller for P(2) than for P(1). However, the purpose of combining the allocation model with the prediction model is not to explain past State-aid patterns. The prediction equation alone was found to be adequate in this regard. The purpose of combining the two models is to enable State level administrators and planners to pose hypothetical questions, to test alternative policies, and

thereby to determine the effect of these various hypotheses and policies on the predicted level of library service in the State. The prediction model completes the information feedback cycle for State level planning.

The allocation model formulated in this chapter has several practical limitations. The sensitivity of the model to changes in the parameters a_i and m, and the difficulties in estimating these parameters, are serious weaknesses for States that would consider immediate adoption of this model. Furthermore, the rather arbitrary allocation of surplus State aid to only one library system, in the absence of a clear policy to distribute the surplus, is another shortcoming. Several recommendations can be made to improve the practicality of the model.

First, constraints could be added to the model to specify further the equitable distribution of State aid. An example is to relate State aid to both the actual level of library support and the potential ability to support library service. If we define λ as a policy variable taking on values between 0.0 and 1.0, then we can specify that State aid can be used to bridge the gap between actual and potential library support. Symbolically,

$$S_i \geq \lambda(Y_i - \pi_i E_i) \quad \text{for } i=1,2,\ldots,22$$

where Y_i = potential library support

$\pi_i E_i$ = actual expenditures by local governments for library service

If $\lambda = 0.0$, then State aid will not be used to make up any fraction of the difference between potential and actual library support. If $\lambda = 1.0$, then State aid will totally bridge the gap. This constraint will only be applicable to systems in which the potential support is greater than actual support. If the quantity $(Y_i - \pi_i E_i)$ is negative, State administrators may wish to include another constraint for the purpose of rewarding those systems whose actual support exceeds their potential support for libraries. Another policy variable α can be defined to accomplish this aim. Symbolically,

$$S_i \geq \alpha(\pi_i E_i - Y_i) \quad \text{for } i=1,2,\ldots,22$$

If $\alpha = 1.0$, then State aid can be used to match dollar for dollar the excess of actual library support over potential library support. Obviously other values for α will offer greater or lesser incentives for increased levels of local support.

A second recommendation for improving the practicality of the model is to formalize the process of distributing surplus State aid. These policies should be explicitly formulated and, if possible, they should be included in the model as further constraints. These additional constraints will preclude any arbitrary assignment of surplus

State aid.

Summary

In this chapter we formulated the allocation process in the framework of a mathematical programming problem. The objectiye function was defined to maximize total library service in the State, where library service was measured by the number of exposure counts. The constraints placed upon the model took into account every library system's ability to pay for library service, local needs for service, and an over-all budget ceiling. The methodology developed in Chapter 3 was used to provide measures of estimated capacity for library support. The output of the model was the optimal amount of State aid allocated to each library system.

The solution of the model was examined for changes in the model's parameters. The first parametric study was for changes in the value of the policy variable, m, representing the cost per capita of providing a minimum acceptable level of library service. Relatively small changes in m resulted in significant changes in the right-hand-side coefficients, and so long as the solution remained feasible, the size of the budget "surplus" was reduced significantly. The second parametric study involved changes in the value of the budget constraint. It

was observed that increases or decreases in the budget resulted in dollar for dollar changes in the size of the "surplus," provided that the solution remained feasible. Thirdly, a parametric study of the objective function coefficients revealed that slight changes in the values of a_i reduced the number of alternate optima, and thereby limited the range of alternative policies that could be formulated to distribute the budget "surplus."

The results of the allocation model then were linked with the prediction equations developed in Chapter 3. The usefulness of this combination was described. The next chapter will discuss extensions of this research.

CHAPTER 5

CONCLUSION

Conclusions

This dissertation addressed the problem of the efficient allocation of State aid for libraries within a fixed budget. Two models were developed: an allocation model to determine the amount of State aid to be received by each library system and a model that predicts library use as a function of State aid and other variables. These models were built, their parameters estimated, and their structure tested and validated. Finally these models were combined for use as a policy-making tool. The detailed results of these efforts are reported in Chapters 3 and 4 and Appendix A.

To summarize the results, the allocation model determined the amount of State aid that should be distributed to each library system in order to maximize a measure of library effectiveness. The allocation process was constrained by an over-all budget ceiling and equity considerations. In the process of specifying the equity constraints, a methodology for constructing measures of ability to pay for library service was devised. These measures were used to estimate the potential capacity of library system areas to support library service. These ability to

pay measures were contrasted with estimates of the cost of meeting a library system's minimum acceptable needs, as defined by State level administrators.

The model was tested using alternative assumptions for the coefficients in the objective function, for several different estimates of the cost of meeting a system's minimum acceptable needs, and for different budgetary ceilings. Under certain conditions the model had multiple solutions. When it had multiple solutions, the model was found to be sensitive to changes in the objective function coefficients. Moderate changes in the cost per capita of meeting a system's needs were shown to have relatively great impact on the equity constraints, and given the size of the current budget, an increase in the value of m from $4.00 to $4.50 per capita was shown to be infeasible. The model was not sensitive to changes in budget.

The results of the prediction model were quite good using data for New York State's system of public libraries. Library use for each system can be predicted very accurately with a multiple regression model that includes only three variables: State aid, change in local government expenditures for libraries, and change in number of books added by local purchase or gifts. It was found that 96.2% of the variation in library use could be explained by these variables. Furthermore, they are mutually independent.

A final result is that the combination of the allocation and prediction models can be quite useful for policy purposes. Alternative policies for distributing State aid were compared, and it was found that the models were biased against urban as opposed to suburban or rural library systems.

Implications for Future Research

The models developed in this dissertation are only representations of reality. Naturally more research is needed to improve the degree to which the models depict the real world. This section identifies several avenues of research which would expand and make more useful the results of this dissertation.

First, the assertion in Chapter 3 that the results of the models can be generalized to other states should be examined carefully. Although the New York pattern of state library development has been followed by other states, differences exist. A useful question to ask is: "What changes are necessary in either the prediction or the allocation model in order to apply them to other states?"

Second, perhaps the scope of the problem should be broadened to include a method for determining the optimal size of the total State library-aid budget. The difficulties inherent in this broader problem are formidible and were discussed briefly in Chapter 3. This approach would

seek to balance the relative costs and benefits of supporting State library programs vis-a-vis all other State programs. The solution to this problem is necessary to lessen the danger of suboptimization in the models presented in this dissertation.

There are at least two estimation problems which deserve further attention. The assumption in Chapter 4 that the coefficients in the objective function of the allocation model are all equal is crude and probably inappropriate. Library systems probably are not equally effective in providing service per dollar of State aid. Research should be conducted on alternative methods for estimating these coefficients, particularly since the model's solution is sensitive to changes in these coefficients. Time series analysis might be helpful if appropriate statistical series were available for a sufficient length of time.

Another estimation problem which should be studied involves the choice of the value for m, the cost per capita of providing a minimum acceptable level of library service in the State. It was shown in Chapter 4 that small changes in the value of m result in relatively large changes in the values of the equity constraint. Disregarding the problems of defining a minimum acceptable level of service, the use of a single value for m is surely an oversimplification. At least one and possibly two dimensions of m should be

recognized. The first dimension should identify the differences in the cost per capita of providing service in systems with varying population sizes. Research is necessary to identify the size classifications for library systems (possibly on the basis of population density) and to measure the differences in cost due to size in providing equivalent levels of library service. The second dimension which could be added would distinguish between the cost of providing library service to different population classes. This is a complex problem, but research should be conducted on measuring the differences in cost in providing equivalent levels of library service to different segments of the population. To illustrate, for a city of a given size, this cost will vary depending on such factors as the educational level of the population. Consideration of these two dimensions would lead to more accurate and realistic estimates of the cost of meeting a library system's minimum needs.

A major extension of this research is the consideration of additional structural factors in the allocation model. The model currently includes only equity constraints and a budget constraint, and it seems to be biased against large urban library systems. Further constraints should be examined for possible inclusion in the model. Examples of other constraints are: (1) accounting for library use in

one system by residents of another system; (2) accounting for the heavy use of strong research libraries (such as the New York Public Library) by residents from other states and by the interlibrary loan network; and (3) using State aid as an incentive tool for particular programs.

In conclusion, rationalization and quantification of allocation and measurement problems of libraries for State level administrators are in an embryonic stage. The models that were developed are important contributions for short-term library planning and decision making. Final decisions about the allocation of State aid and final choices among alternative policies will not be made on the basis of these models since other variables not included in this research (such as political trade-offs) certainly need to be considered. However, the general conclusion of this dissertation is that these models should form the basis for rational library planning and decision making at the State level.

APPENDIX A

STATISTICAL ANALYSIS OF THE PREDICTION EQUATION

This appendix provides a detailed description of the data and a detailed discussion of the statistical results that are summarized in Chapter 3. The data for New York State's 22 library systems are presented in Table A.1. The symbols are defined in the glossary.

The principal source for the data in Table A.1 is Public and Association Libraries Statistics/1968, The University of the State of New York, New York State Library, Division of Library Development. This annual publication of New York State library statistics is compiled from the reports of individual libraries. These reports are a prerequisite for State aid. The data for the 1967 report were not published, although they were made available to the author in rough form. The 1967 data were used to derive two of the variables described below, ΔG and ΔK. The variables in Table A.1 shall now be described.

The variable C is the annual number of circulations recorded for each library system in 1968, measured in units of 100,000 circulations. Photocopies provided in lieu of circulation should be included here, as well as the loan of each book, serial, or nonbook item.

The variable D is the population density of the area

Table A.1

Data Base for Prediction Equation Results, New York State Library Systems

System	C[a]	D[b]	LnD[b]	G[c]	ΔG[c]	N[d]	ΔN[d]	H[e]	M[f]	O[c]	S[f]
BN	82.11	34,235	10.44	72.74	- .19	335.27	-20.58	301.97	159.28	97.01	160.51
BE	64.78	1,056	6.96	46.85	2.43	200.85	-24.98	244.34	107.93	55.31	86.56
CC	13.19	95	4.55	3.65	.24	41.61	- 7.45	52.54	18.45	7.83	24.34
CST	12.87	80	4.38	4.21	.17	48.35	- 4.12	73.45	19.38	9.05	28.10
CEF	6.47	33	3.50	1.29	.12	18.93	- .20	31.68	4.52	3.92	20.92
FL	11.02	111	4.71	2.78	.21	31.49	- 1.88	46.57	10.90	9.22	31.68
FC	15.22	90	4.50	8.32	.67	69.42	7.41	78.36	25.15	13.23	41.33
MH	13.75	164	5.10	5.41	.57	52.35	11.73	64.81	24.27	13.27	35.65
MY	15.15	121	4.80	7.40	.81	69.04	7.22	73.52	27.75	12.88	34.65
MV	13.40	169	5.13	5.72	.04	36.43	2.30	54.11	14.21	10.46	26.62
NAS	99.50	5,669	8.64	89.20	7.09	321.69	-67.47	330.81	176.99	99.98	103.70
NY	121.95	26,338	10.18	116.92	1.89	564.49	68.09	508.31	335.29	237.95	288.89
NIO	13.85	232	5.45	7.81	1.36	52.71	-15.57	61.20	22.98	11.66	32.33
NC	11.69	51	3.93	4.12	.24	34.34	- 5.97	68.74	14.83	8.05	42.26
O	23.60	556	6.32	10.08	.25	56.76	- 5.24	61.62	28.64	13.21	29.40
P	49.00	304	5.72	25.48	4.03	141.50	9.34	159.53	63.55	35.89	79.39
Q	81.15	17,466	9.77	73.78	1.06	269.90	3.85	240.54	151.46	97.72	137.31
RAM	19.72	208	5.34	15.69	1.30	117.86	24.70	79.59	47.86	20.09	46.07
SA	8.72	53	3.97	2.59	.46	37.25	.62	43.65	9.69	5.77	29.21
SU	49.07	1,211	7.10	39.70	6.63	225.36	11.41	156.24	102.43	55.69	70.26
UH	13.82	366	5.90	7.03	.36	49.89	- 7.54	59.61	20.26	10.34	26.63
W	65.21	2,049	7.63	45.30	9.06	198.77	37.12	187.42	75.58	56.68	60.72

a measured in units of 100,000
b measured in actual units
c measured in units of $100,000
d measured in units of 1,000
e measured in units of 10,000
f measured in units of $10,000

served by each library system. The values for D are derived by dividing the population of each library system (1970 Census data were used) by the area in square miles of the system. Since intercensal population estimates are not readily available for counties, it was felt that 1970 data were more appropriate for calculating D than 1960 data. The values for D are in actual units. The variable LnD is the natural logarithm of D.

The variable G is the dollar value in units of $100,000 of the receipts into the library system's operating fund from local public funds in 1968. Included in G are any monies received from the village, town, city, school district, and county, as well as any monies received as a result of contractual agreements between the library system and these taxing authorities. The variable ΔG is the annual rate of change of G. It is calculated for each system by taking the difference in the value of G for 1968 and the value for G in 1967.

The variable K is the number of books added by local purchases and gifts to the book stock in 1968, measured in units of 1,000. It includes both adult and juvenile books, but it does not include periodicals, audio-visual materials, materials for the blind, or other nonbook resources. Also excluded are books added through system rotating collections or bulk loans. The variable ΔK is the annual rate of

change of K. It is calculated for each system by taking the difference between the value for K in 1968 and its value in 1967.

The variable H represents the total book holdings for each library system in 1968, measured in units of 10,000 books. It includes both adult and juvenile books to which the system retains title, and all materials given to the system under State and Federal book programs.

The variable M represents the amount of expenditure from the operating fund for library materials in 1968 measured in units of $10,000. These expenditures are used for books, serials, audio-visual materials, materials for the blind, other nonbook materials, and binding.

The variable O represents total operating fund expenditures for each system in 1968, measured in units of $100,000. The value for this variable includes expenditures for library materials in addition to expenditures for salaries, employee benefits, library services and supplies (e.g. office supplies, telephone, postage, and travel), buildings and equipment operation and maintenance, and automotive equipment (e.g. bookmobile).

The variable S represents the amount of State aid received by each library system in 1968, measured in units of $10,000.

The results of the regression of C on selected sets

of these variables are shown in Table A.2. These results will be discussed, as well as the results of various statistical tests performed on the data.

Initially, the two variables D and G were studied as possible exogenous variables in the prediction model. The relationship between C and D is clearly not linear, as described in Chapter 3. Equation (1) of Table A.2 indicates that the least squares fit of the data has an intercept value of 25.3783 and a slope of .0027 for each unit change in D. In this and subsequent equations, the values in parentheses below each equation are the estimated standard errors of the regression coefficients. In this case the estimated standard errors for the intercept term and the slope coefficient are 5.5216 and .0006 respectively. The ratio of a regression coefficient to its standard error is a t statistic, and the significance of the estimate can be found by comparing this computed ratio with the appropriate critical value of the t distribution. Small values of the ratio imply that the coefficient is not significant. The t statistic for the coefficient of D is equal to 4.500. At the .01 level the critical t value for 20 degrees of freedom is 2.845. Thus, we conclude that this coefficient is significant at the .01 level.

Three summary statistics are also provided for each equation: $\bar{R}^2$, S_e, and F. $\bar{R}^2$ is the coefficient of deter-

Table A.2

Regression Results for Development of Prediction Equation

Regression Equation	$\bar{R}^2$	S_e	F
(1) C= 25.3783 + .0027 D (5.5216) (0.0006)	.527	23.61	24.43
(2) C= -56.9411 + 15.3486 LnD (8.9430) (1.3925)	.852	13.23	121.49
(3) C= 9.0163 + 1.0181 G (1.3989) (0.0330)	.978	5.09	952.04
(4) C= 4.1046 + .2404 K (2.7406) (0.0144)	.930	9.08	280.47
(5) C= 9.8885 + .1793 H (4.4009) (0.0196)	.798	15.45	83.75
(6) C= 9.4505 + .4087 M (3.0172) (0.0294)	.901	10.78	192.99
(7) C= 13.0987 + .5841 O (3.5167) (0.0527)	.853	13.16	122.93
(8) C= 4.3007 + .4915 S (4.8413) (0.0539)	.797	15.49	83.23
(9) C= 6.2496 + .7857 LnD + 1.0020 G + .0262 K - .0779 S (7.0154) (1.4738) (0.2092) (0.0627) (0.0705)	.977	5.18	226.70
(10) C= -20.2926 + 4.2937 LnD + 3.7553 ΔG - .2256 ΔK + .3702 S (5.3276) (1.2795) (0.5039) (0.0557) (0.0423)	.976	5.38	209.63
(11) C= -3.4752 + 4.4629 ΔG - .3093 ΔK + .4948 S (2.2651) (0.5817) (0.0624) (0.0254)	.962	6.74	175.63

mination, adjusted for degrees of freedom. This value indicates the proportion of the total variation in C which is explained by the regression line. An adjustment is necessary because of the negative bias of the unadjusted estimator, R^2. In Equation (1), the value $\bar{R}^2 = .527$ indicates only a moderately good fit of the relationship. The quantity S_e is the standard error of estimate. This is a measure of the extent of the variability of the estimated values of the dependent variable along the sample regression plane. In Equation (1), $S_e = 23.61$ indicates a fairly high degree of variability of the regression relationship, a reflection of the rather loose fit of the relationship. The last statistic, F, is the value for the analysis of variance test of the over-all significance of the regression relationship. It is computed from the unadjusted value, R^2, by the formula:

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

where k is the number of independent variables and n is the number of observations. In all these formulas, n = 22. In Equation (1), the value F = 24.43 can be used to test the significance of the regression relationship by comparing this quantity with the appropriate critical value of the F distribution. Large values of F imply a significant relationship; small values, an insignificant relationship. In

this case, the critical F value at the .01 level with 1 degree of freedom (d.f.) in the numerator and 20 d.f. in the denominator is F(1,20) = 8.10. Since 24.43 is greater than 8.10, we conclude that the regression relationship is significant at the .01 level.

The transformation of D that provides the best linearization of the relationship between C and D is LnD, the natural logarithm of D. Equation (2) shows the results of regressing C on LnD. Comparing this equation with Equation (1), we can readily see the improvement: $\bar{R}^2$ increases from .527 to .852, S_e decreases from 23.61 to 13.23, and the F value increases from 24.43 to 121.49. The t statistic for the intercept has increased from 4.596 to -6.367, and for the slope from 4.500 to 11.022. LnD is clearly superior to D as a predictor of C.

The relationship between C and G is very strong, as indicated by Equation (3). Of all the variables studied, this variable is the single best predictor of C. The t statistic for the coefficient of G has a value of 30.851. At the .01 level the critical t value for 20 d.f. is 2.845. Since the computed value greatly exceeds the critical value, we very strongly reject the hypothesis at the .01 level that there is no relationship between C and G.

Equations (4), (5), (6), and (7) show the results of regressing C on the four variables subject to control at or below the library system level. The best predictor of

C in this group (ranked in order of $\bar{R}^2$) is K, and is followed in order by M, O, and H. For this group, the regression of C on K yields the highest value of $\bar{R}^2$ (.930), the smallest value of S_e (9.08), and the highest F value (280.47).

We can examine the net additional effect of including either M, O, or H with K in a regression relationship by calculating the increment to R^2. We can test the significance of this increment using an F test. The test statistic is calculated by the formula:

$$F = \frac{\Delta R^2/\ell}{(1-R^2)/(n-k-1)}$$

where ΔR^2 is the increment to R^2 due to the additional variable(s)

ℓ is the number of variables being added (in this case, $\ell = 1$)

R^2 is the coefficient of determination when all k variables are included

n is the number of observations ($n = 22$)

k is the number of variables under consideration (in this case, $k = 2$)

Table A.3 presents the results. At the .01 level, the critical value $F(1,19) = 8.18$ exceeds the computed F values for the increment to R^2 of each of the three variables. Thus none of them contributes significantly to the predictive power of K alone, and they will be excluded from fur-

Table A.3

Increment to R^2 Due to Addition of M, O, and H in Multiple Regression of C on K .

Source	R^2	ΔR^2	F value
Addition of M	.935	.0016	0.47
Addition of O	.940	.0067	2.12
Addition of H	.940	.0069	2.19

ther analysis.

Equation (8) shows the results of the regression of C on S. As indicated by $\bar{R}^2$ = .797 and F = 83.23, the relationship is quite strong. The hypothesis that the regression coefficient of S is equal to zero is rejected at the .01 level since the computed t value of 9.119 exceeds the critical t value for 20 d.f. of 2.845.

The analysis at this point has identified four variables for inclusion in the prediction model: LnD, G, K, and S. The multiple regression results of these four independent variables and the dependent variable C are shown in Equation (9). Although the summary statistics are good, $\bar{R}^2$ = .977, S_e = 5.18 and F = 226.70, the standard errors of the coefficients are high. This results is low values for the associated t statistics, as shown below. At the .01 level the critical t value for 17 d.f. is 2.898.

Coefficient	t value
constant	.891
LnD	.533
G	4.790
K	.418
S	-1.105

The low values for t indicate that the independent variables may be collinear, a suspicion more strongly confirmed by examining the pairwise correlation coefficients

of these variables. These coefficients, presented in Table A.4, range in size from .84 to .98. These high values suggest that the multicollinearity problem in the data is severe.

Multicollinearity can be detected statistically by a series of tests of independence among the explanatory variables. The hypotheses under test are that each variable in the group is statistically independent of the other three variables. The procedure for conducting the tests is to calculate the R^2 value for each relationship, beginning with the regression of LnD on S, G, and K. The significance of the relationship can be judged by computing the F statistic used earlier:

$$F = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

and comparing this value with the critical F value with k and (n-k-1) d.f. The results of these tests are shown in Table A.5. All four computed F values in the table are greater than the critical calue F(3,18) = 5.09 at the .01 level. Thus we reject all four hypotheses of independence and conclude that at the .01 level all variables are collinear.

The problem of multicollinearity can be partially resolved if we define ΔG as the annual rate of change in G and ΔK as the annual rate of change in K. Regressing C

Table A.4

Correlation Matrix for LnD, G, K, and S

	LnD	G	K	S
Ln D	1.00	.93	.90	.84
G		1.00	.98	.92
K			1.00	.96
S				1.00

Table A.5

Test Results for the Presence of Multicollinearity: LnD, S, G, and K

Dependent Variable	Explanatory Variables	R^2	F-value
LnD	S, G, and K	.863	37.88***
S	LnD, G, and K	.935	86.00***
G	LnD, S, and K	.974	223.09***
K	LnD, S, and G	.983	346.06***

*** Significant at the .001 level.

on LnD, ΔG, ΔK, and S has little effect on the over-all results, as shown in Equation (10). A comparison of this equation with Equation (9) shows that $\overline{R}^2$ is practically unchanged, S_e has increased from 5.18 to 5.38, and the F value has decreased from 226.70 to 209.63. Improvement is indicated, however, in the relative size of the standard errors of the coefficients. Comparing the relevant t statistics in Equation (9) and Equation (10), the only significant t statistic at the .01 level in Equation (9) corresponds to the coefficient for G. This t value equals 4.790. In Equation (10), every t statistic is significant at the .01 level, as shown below:

Coefficient	t value
constant	-3.809
LnD	3.356
ΔG	7.365
ΔK	-4.053
S	8.752

At the .01 level the critical t value for 17 d.f. is 2.898.

An examination of the correlation matrix for the respecified model is shown in Table A.6. With one exception, all the pairwise correlation coefficients are low, falling within the range of .02 to .38. The exception is the correlation coefficient of .84 between LnD and S.

The results of tests for the presence of multicolline-

Table A.6

Correlation Matrix for LnD, ΔG, ΔK, and S

	LnD	ΔG	ΔK	S
LnD	1.00	.38	.06	.84
ΔG		1.00	.02	.20
ΔK			1.00	.33
S				1.00

arity are shown in Table A.7. Similar to the preceding tests, we are testing the hypotheses that each variable is independent of the other three variables. We are using R^2 to calculate the test statistic F. The critical F value at the .01 level is F(3,18) = 5.09. In two of the four cases, the computed F value exceeds the critical value. We conclude that LnD and S are collinear with the set of other variables, while ΔG and ΔK are independent of the other variables.

This mixed result can be fully resolved if we eliminate LnD from the model. Tests for multicollinearity among the set of variables S, ΔG, and ΔK yield the results shown in Table A.8. At the .01 level the critical value is F(2,19) = 5.93. Under the null hypotheses, all three computed F values are less than the critical value. Thus we conclude that S, ΔG, and ΔK are mutually independent.

The results of the regression of C on S, ΔG, and ΔK are shown as Equation (11) of Table A.2. This equation indicates a strong linear relationship: $\bar{R}^2 = .962$, $S_e = 6.74$, and F = 175.63. These results compare favorably with the results of the model which includes LnD. The greatest improvement in the reduced model is the increased accuracy of the estimates. Except for the constant term, every coefficient in the reduced model has an associated t value that is greater than the corresponding value of the

Table A.7

Test Results for the Presence of Multicollinearity: LnD, S, ΔG, and ΔK

Dependent Variable	Explanatory Variables	R^2	F-value
LnD	S, ΔG, and ΔK	.804	24.63***
S	LnD, ΔG, and ΔK	.804	24.65***
ΔG	LnD, S, and ΔK	.207	1.57
ΔK	LnD, S, and ΔG	.288	2.43

*** Significant at the .001 level

Table A.8

Test Results for the Presence of Multicollinearity: S, ΔG, and ΔK

Dependent Variable	Explanatory Variables	R^2	F-value
S	ΔG and ΔK	.147	1.64
ΔG	S and ΔK	.043	.43
ΔK	S and ΔG	.110	1.18

previous model. These t values are:

Coefficient	t value
constant	-1.534
ΔG	7.672
ΔK	-4.957
S	19.490

At the .01 level, the critical t value for 18 d.f. is 2.878. The most significant of these changes is the increase in the t value for the coefficient of S from 8.752 to 19.490 by eliminating LnD from the prediction model.

The final form of the prediction model is the equation:

$$C = \underset{(2.2651)}{-3.4752} + \underset{(0.5817)}{4.4629}\,\Delta G - \underset{(0.0624)}{.3093}\,\Delta K + \underset{(0.0254)}{.4948}\,S$$

$$\bar{R}^2 = .962$$

$$S_e = 6.74$$

The analysis of the prediction equation is almost complete. The last procedure we shall use to validate the relationship between C and the independent variables is to plot the predicted values of C, denoted by $\hat{C}$, against the residuals. The residuals are calculated by taking the difference between C and $\hat{C}$ for each library system. This plot, shown in Figure A.1, indicates that the variation around the regression plane does not seem to depend on the magnitude of $\hat{C}$. There are 22 values for the residuals: 10 negative and 12 positive. Fourteen values are clustered

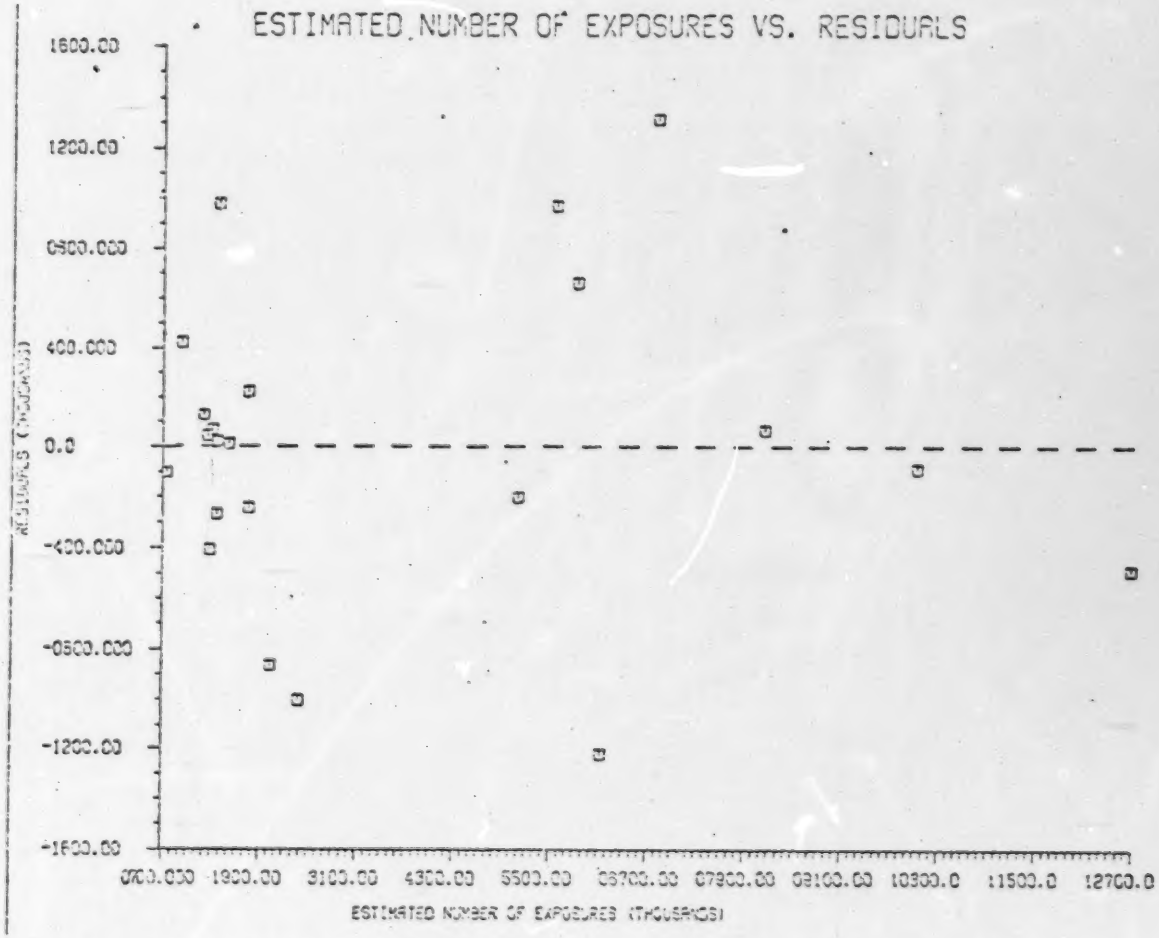

Figure A.1: Plot of Estimated Number of Exposures against Prediction Equation Residuals

below the value $\hat{C} = 3100$. Of these, 6 are negative and 8 are positive. Of the remaining 8 residuals corresponding to values of $\hat{C}$ greater than 3100, 4 are negative and 4 are positive. This plot supports the validity of the prediction equation. The analysis of the model is now completed.